# Basic External Memory Data Structures

Zorieh Soltani

Yazd University

Fall-1389

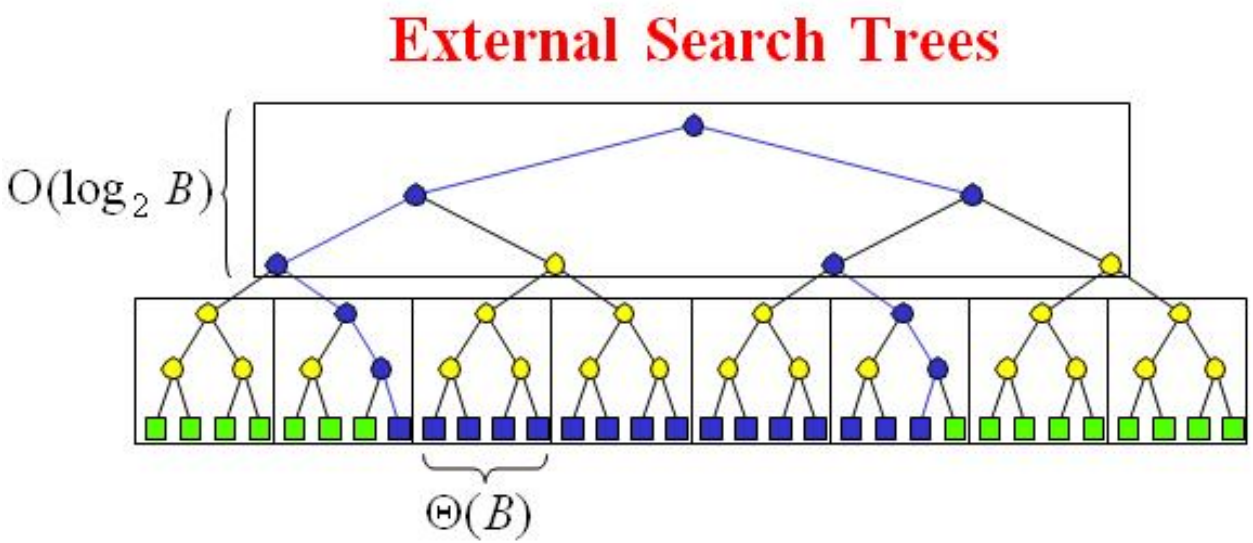
# Content

- 2.3 B-trees
- 2.4 Hashing Based Dictionaries
- 2.5 Dynamization Techniques

- B-trees

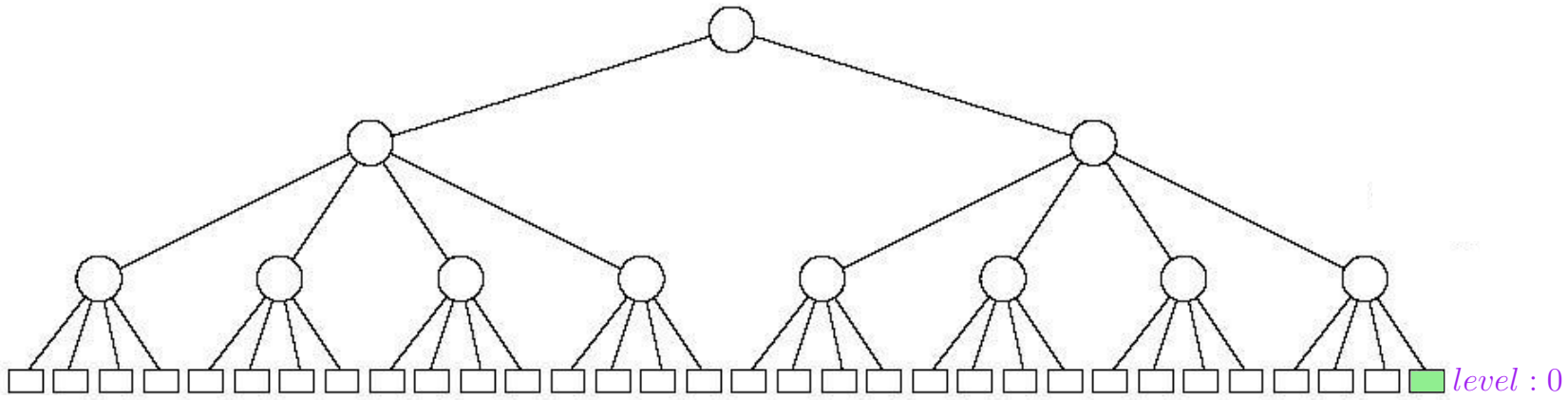
# Introduction

- We want search trees of **large degree** because of using all the information we get when reading a block to guide the search
- B-trees are a generalization of balanced binary search trees to balanced trees of degree $\Theta(B)$
- N: the size of the key set and B: the number of keys or pointers that fit in one block



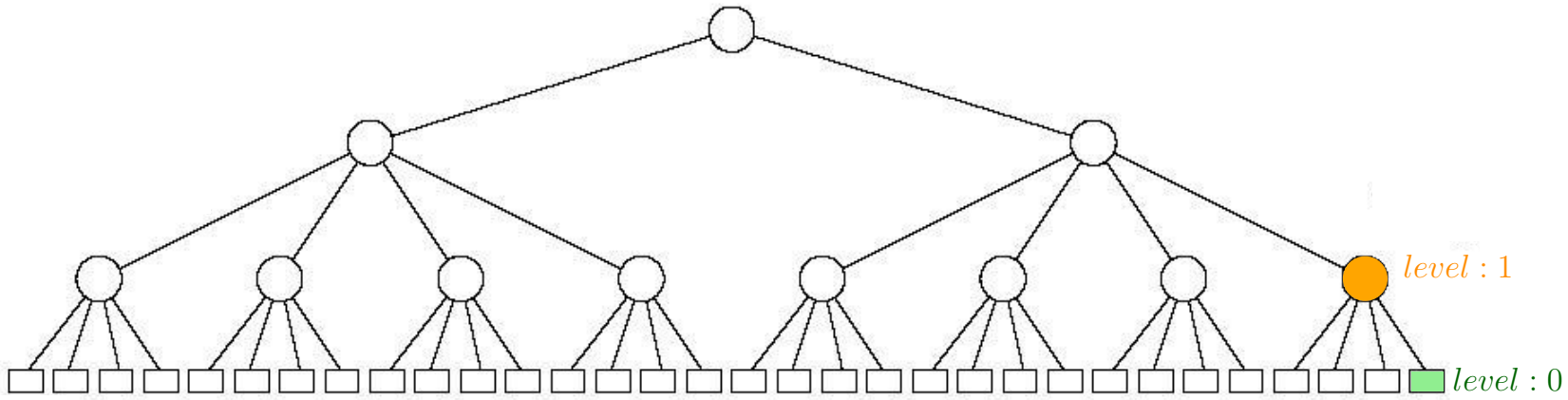
# Introduction(continue)

- In a B-tree all leaves have the same distance to the root
- *Level* of a node: its distance to its descendant leaves
- Weight of node v: the number of leaves subtree of node v,is shown by w(v)



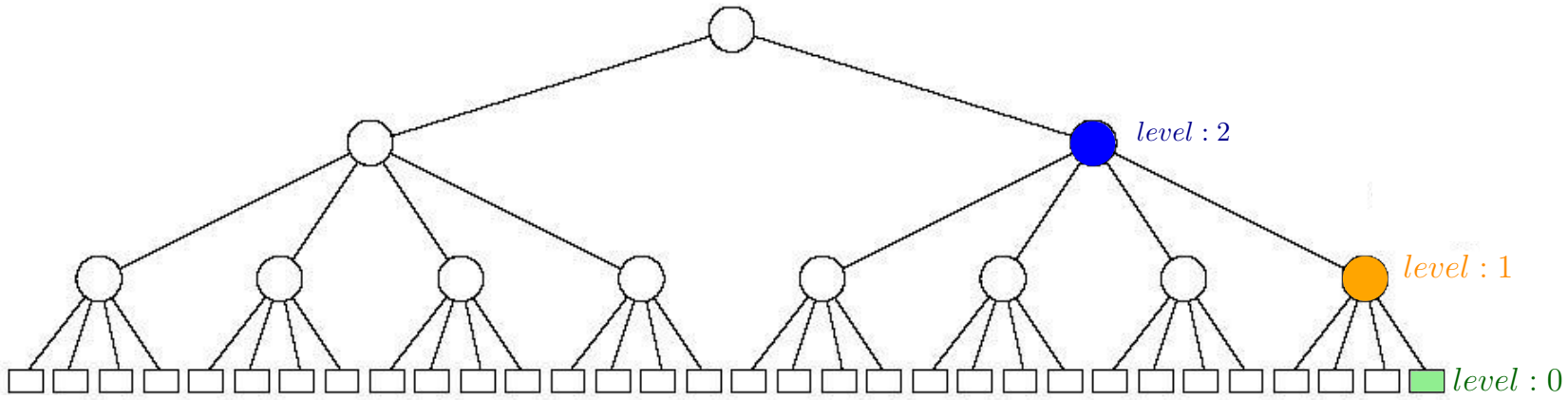
# Introduction(continue)

- In a B-tree all leaves have the same distance to the root
- *Level* of a node: its distance to its descendant leaves
- Weight of node v: the number of leaves subtree of node v,is shown by w(v)



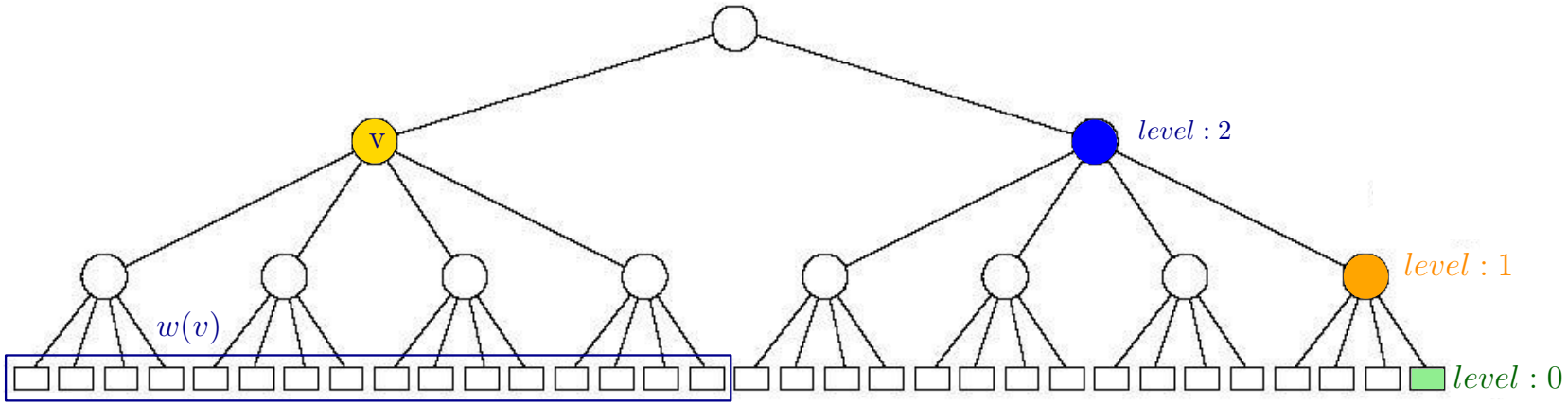
# Introduction(continue)

- In a B-tree all leaves have the same distance to the root
- *Level* of a node: its distance to its descendant leaves
- Weight of node v: the number of leaves subtree of node v,is shown by w(v)



# Introduction(continue)

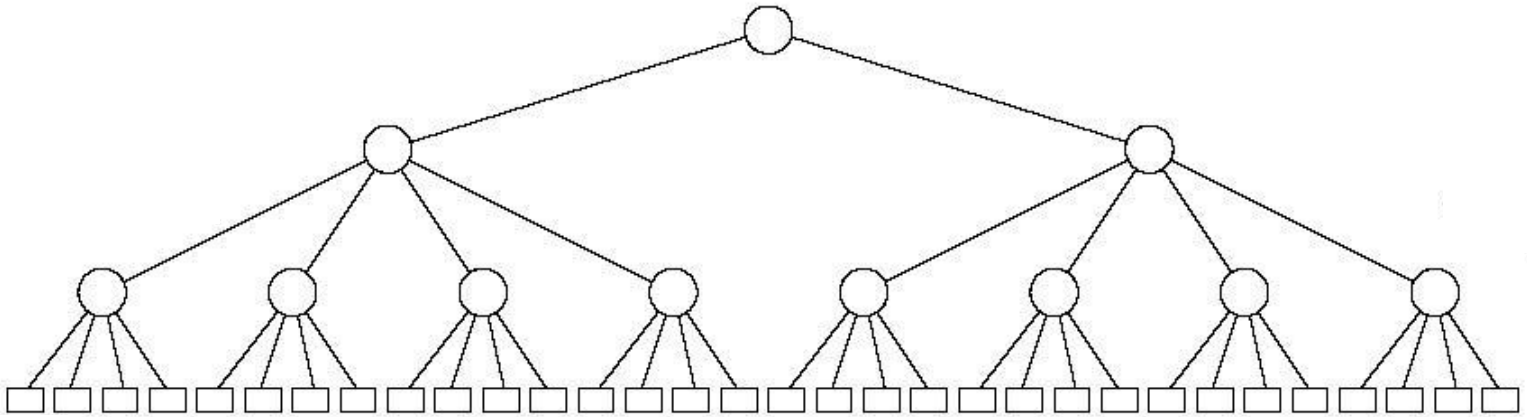
- In a B-tree all leaves have the same distance to the root
- *Level* of a node: its distance to its descendant leaves
- Weight of node v: the number of leaves subtree of node v,is shown by w(v)



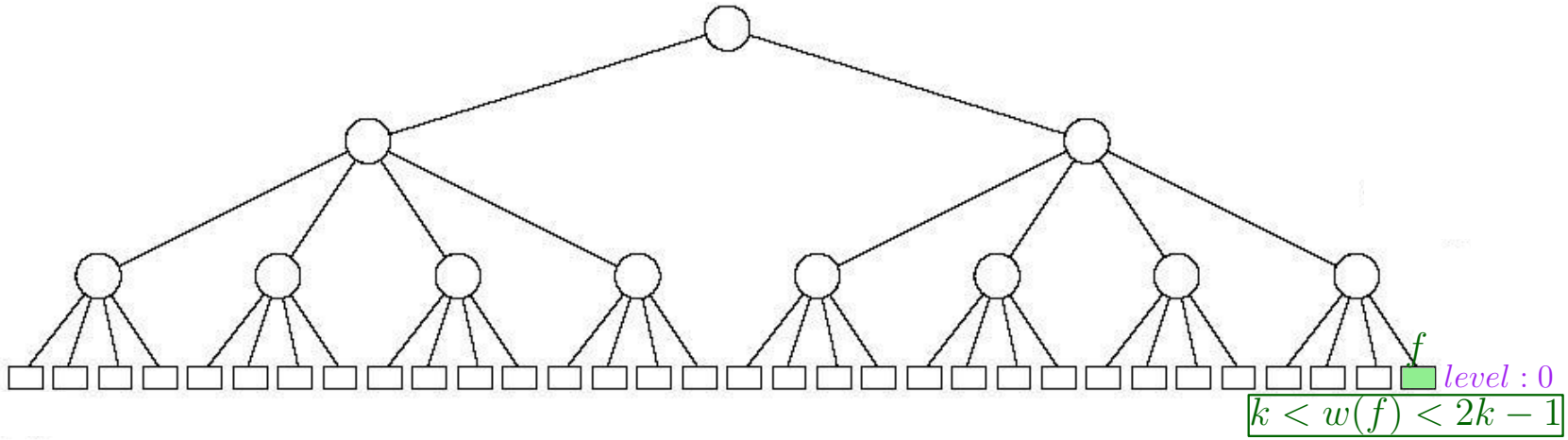
# Definition

T is a weight-balanced B-tree with branching parameter **b** and leaf parameter **k**,($b \geq 4$ and $k \rangle\ 0$ )if:

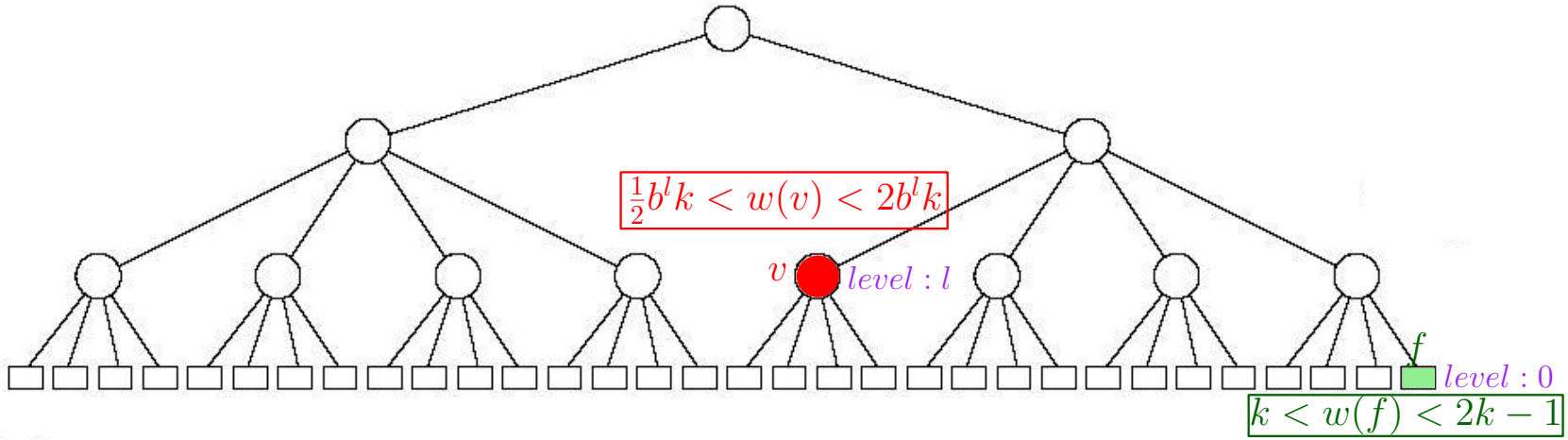
- All leaves of T have the same depth and weight between k and $2k-1$
- An internal node on level l has weight less than $2b^l k$
- An internal node on level l except for the root has weight greater than $\frac{1}{2}b^l k$
- The root has more than one child



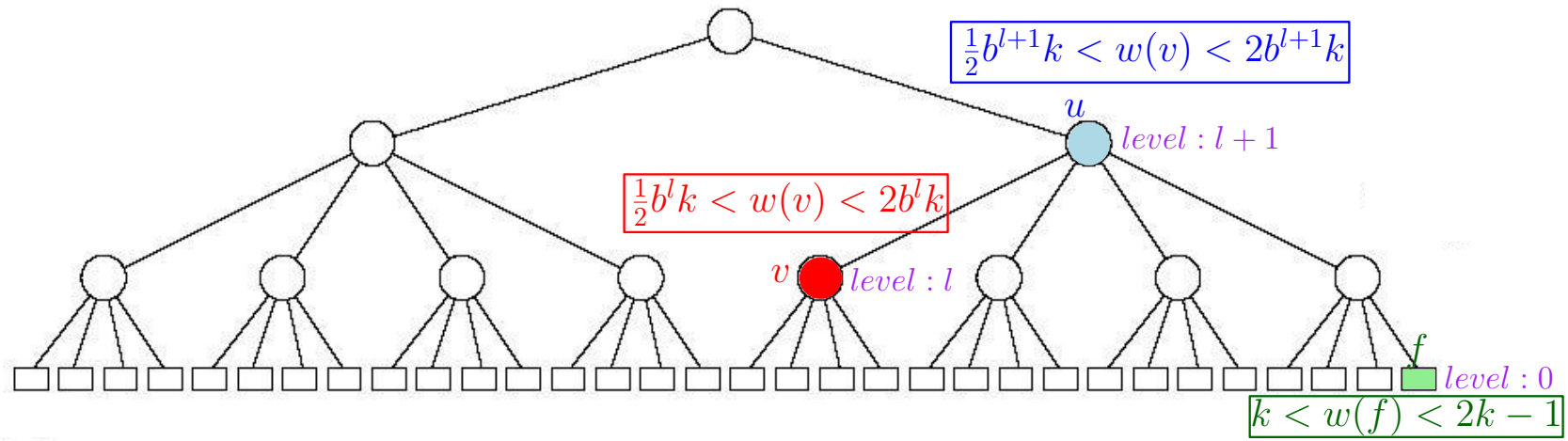
- Limitation on weight results Limitation on degree of each node



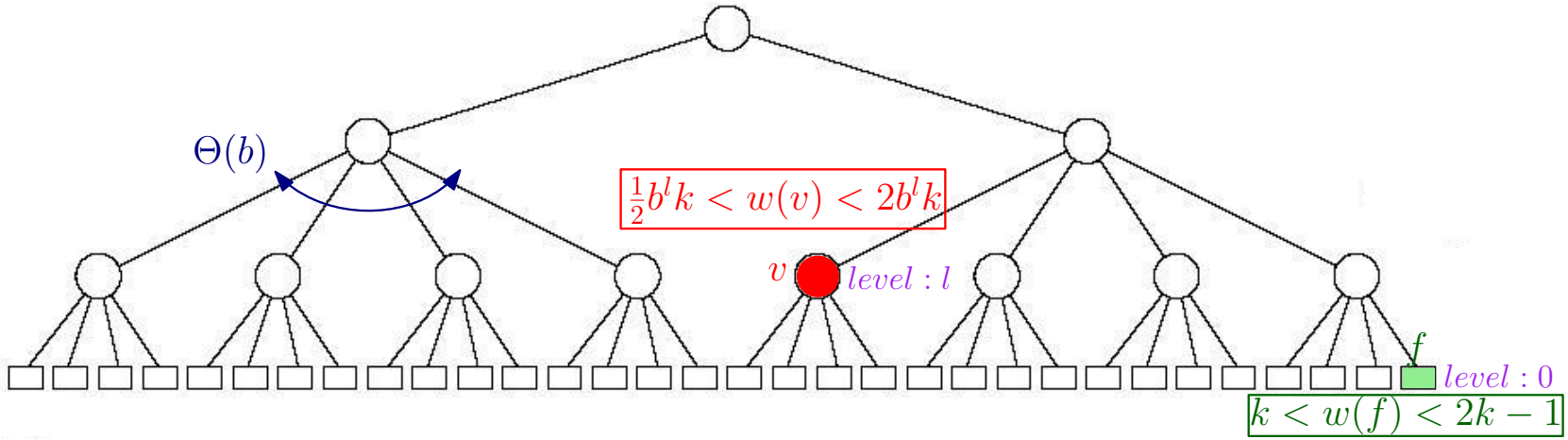
- Limitation on weight results Limitation on degree of each node



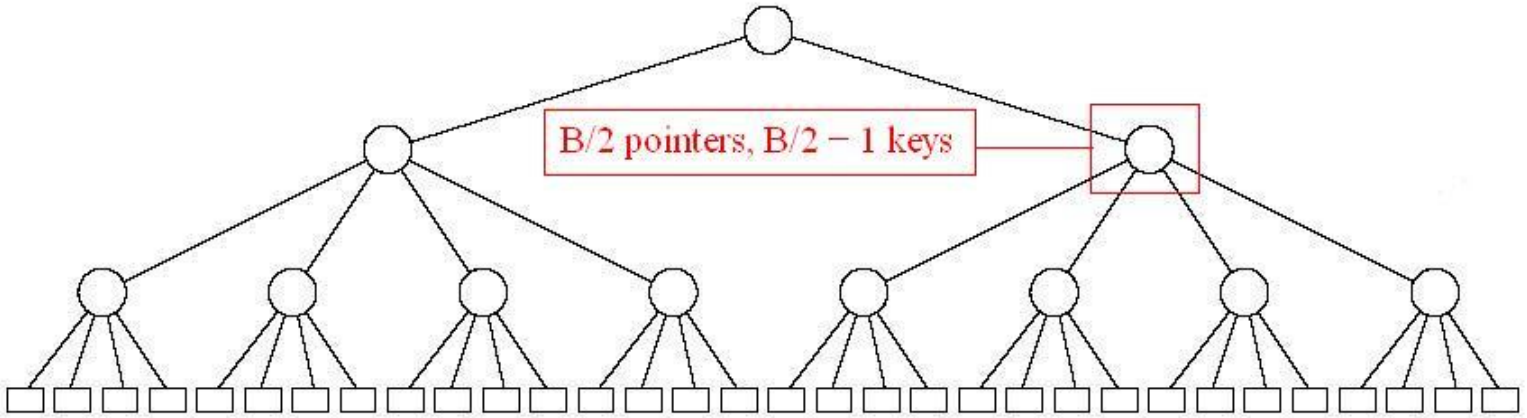
- Limitation on weight results Limitation on degree of each node
- Degree of each node is between $\frac{b}{4}$ and 4b



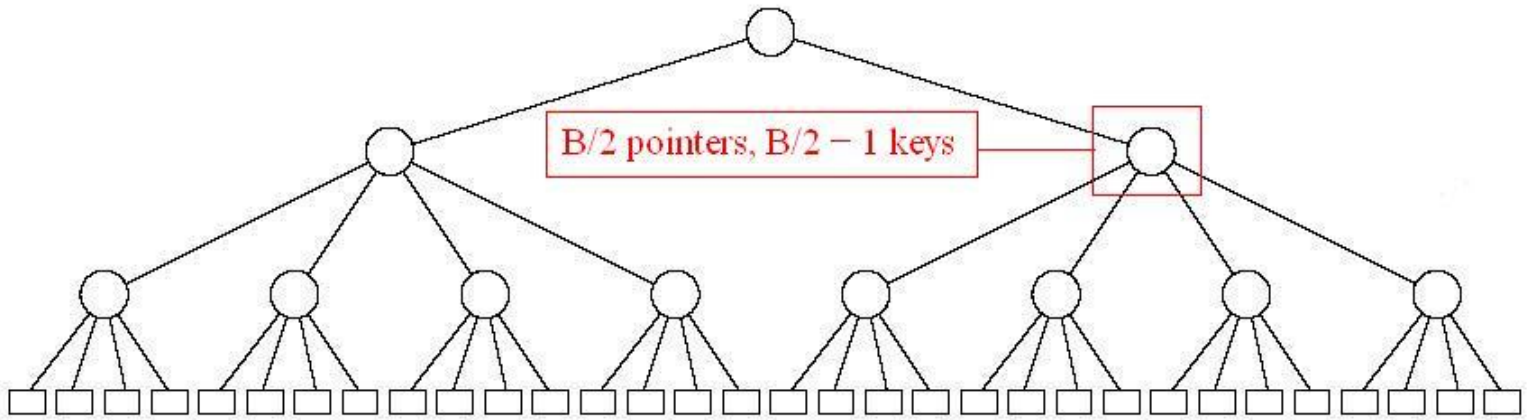
- Limitation on weight results Limitation on degree of each node
- Degree of each node is between $\frac{b}{4}$ and 4b
- The degree of any non-root node is $\Theta(b)$



# The New B-tree is introduced by our book



# The New B-tree is introduced by our book




## The Result

The result branching parameter is: $\mathbf{b} = \frac{B}{8}$

And we assume leaf parameter: $\mathbf{k} = 2$

# The New B-tree is introduced by our book (continue)

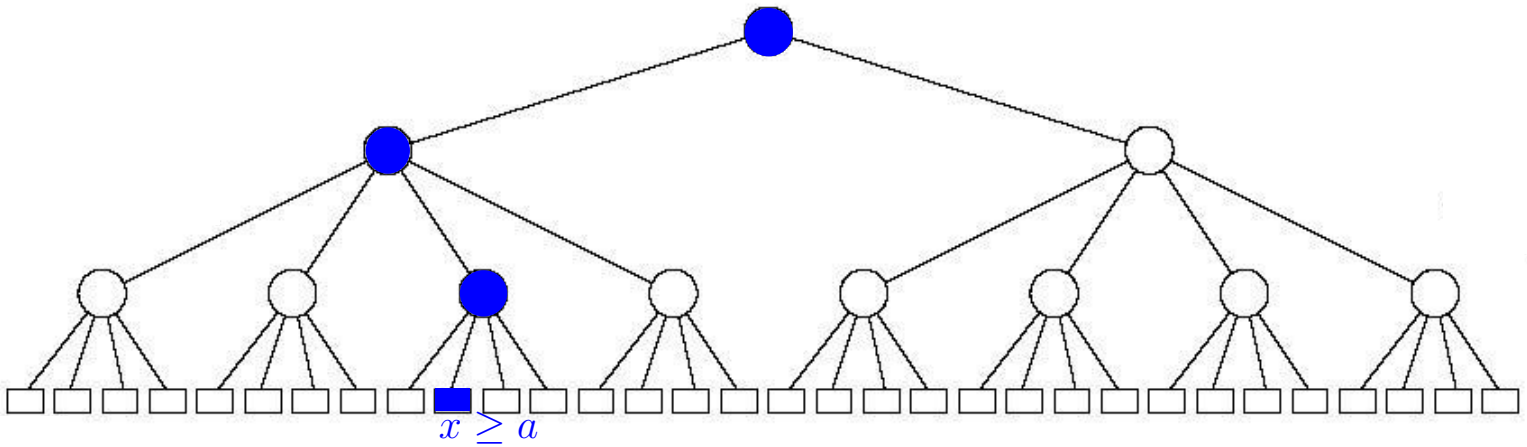
- An internal node on level i has weight less than $4(\frac{B}{8})^i$
- An internal node on level i except for the root has weight greater than $(\frac{B}{8})^i$
- Any node has less than B/2 children
- Any non-root node has greater than B/32 children

# Searching a B-tree

- In a node v stores sorting keys $k_1, ..., k_{d_v-1}$
- The ith subtree of v stores keys k with $k_{i-1} \leq k < k_i$ (defining $k_0 = -\infty$ and $k_{d_v} = \infty$).
- the information in a node suffices to determine in which subtree to continue a search
- The worst-case number of I/Os needed for searching a B-tree equals the worst-case height of a B-tree, at most $1 + \lceil log_b^N \rceil$

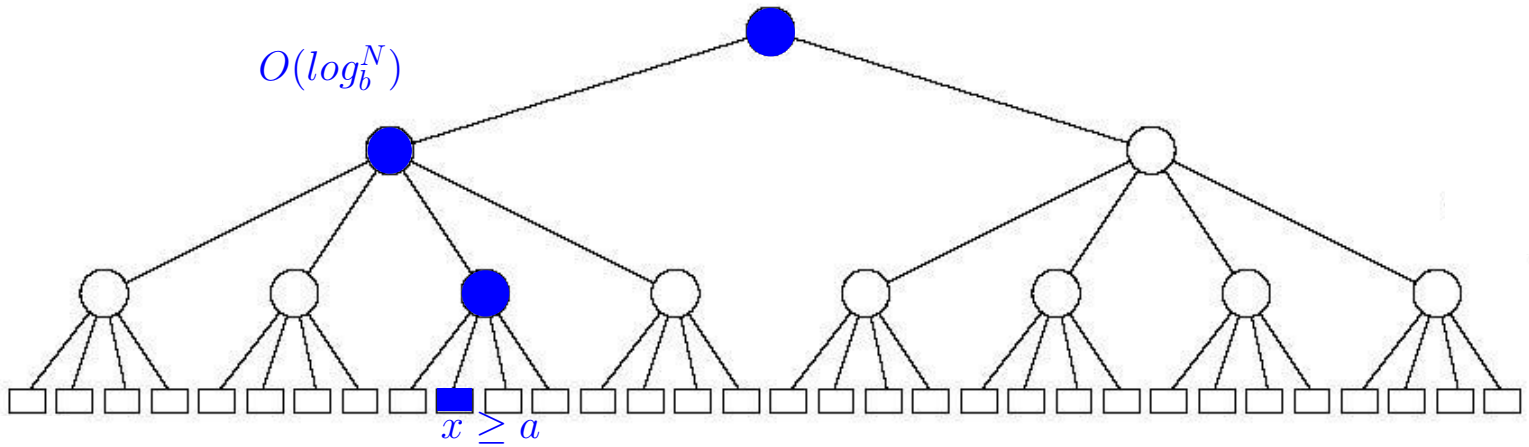
"report all keys in the range [a,b]"

- Search for the key a, which will lead to the smallest key x $\geq$ a
- Traverse the linked list starting with x and report all keys smaller than b
- of I/Os of Rang queries(output sensitivity): $O(log_b^N + Z/B)$



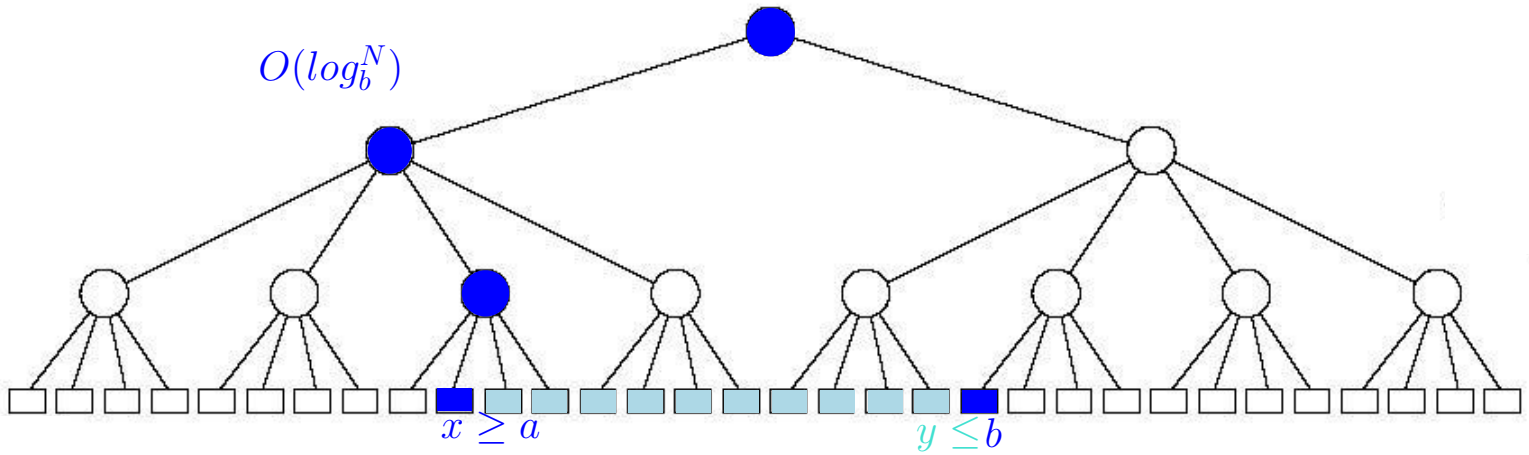
## "report all keys in the range [a,b]"

- Search for the key a, which will lead to the smallest key x $\geq$ a
- Traverse the linked list starting with x and report all keys smaller than b
- of I/Os of Rang queries(output sensitivity):$O(log_b^N + Z/B)$



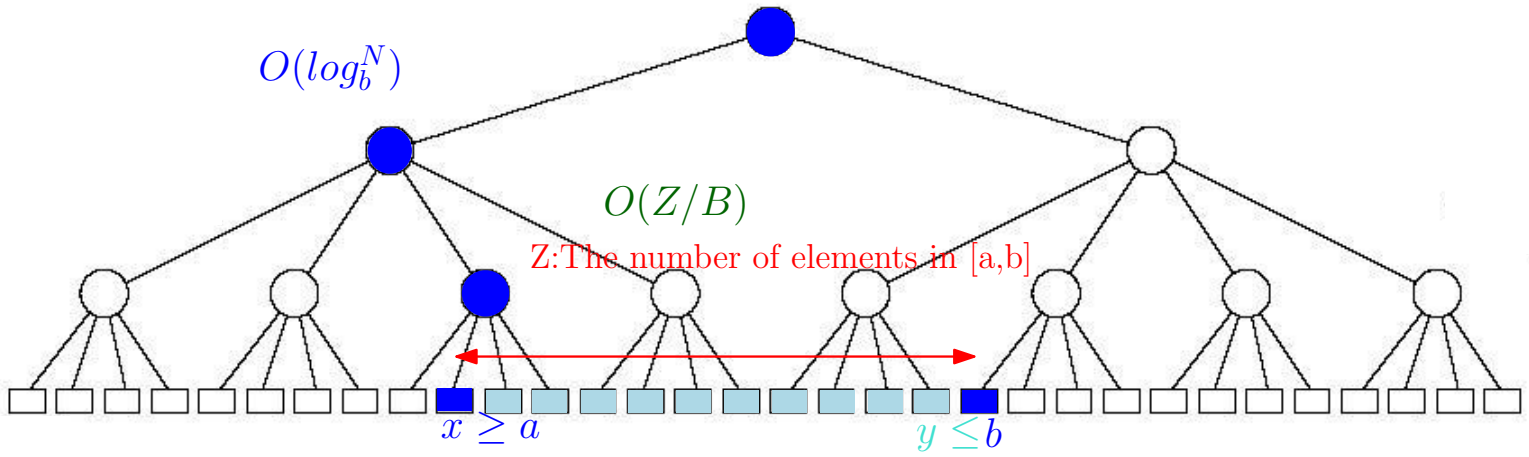
## "report all keys in the range [a,b]"

- Search for the key a, which will lead to the smallest key x $\geq$ a
- Traverse the linked list starting with x and report all keys smaller than b
- of I/Os of Rang queries(output sensitivity):$O(log_b^N + Z/B)$



"report all keys in the range [a,b]"

- Search for the key a, which will lead to the smallest key x $\geq$ a
- Traverse the linked list starting with x and report all keys smaller than b
- of I/Os of Rang queries(output sensitivity): $O(log_b^N + Z/B)$



# Range Reporting(continue)

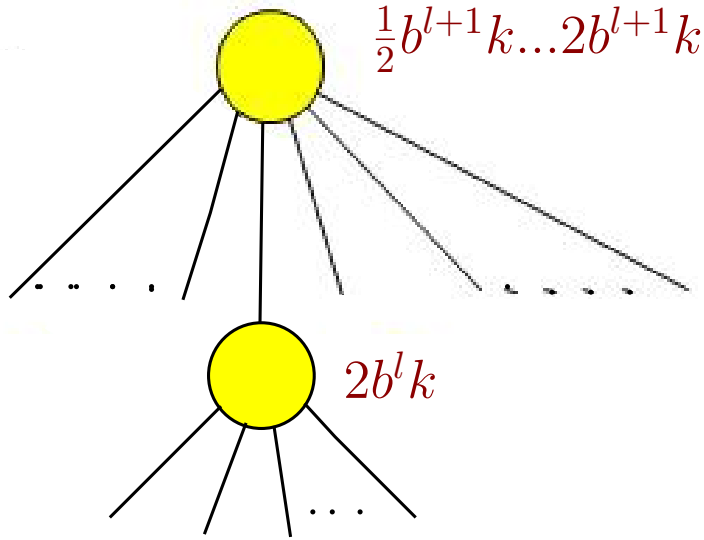
## Two Notes

1. Optimal solution is based on hashing data structures that performs in $O(1 + Z/B)$
2. Optimal output sensitivity fails when query changes to "report the first Z keys in the range [a,b]"

# Inserting and Deleting Keys in a B-tree

## Inserting Key x

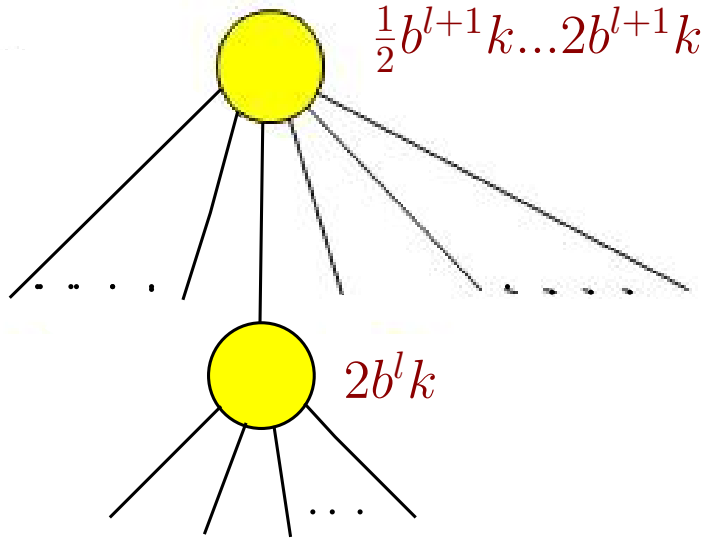
- Search for the key x, find node v that is parent of x
- Insert the key x to node v
- If at level i, w(v)=$2b^l k$ (overweight), we rebalance it by "split"
- We split a node v to two new nodes u,u'
- starting from the bottom and going up



# Inserting and Deleting Keys in a B-tree

## Inserting Key x

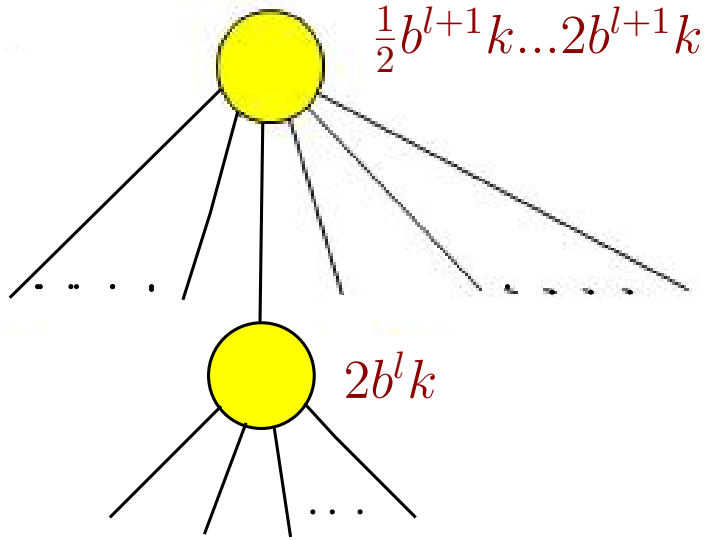
- Search for the key x, find node v that is parent of x
- Insert the key x to node v
- If at level i, w(v)=$2b^lk$ (overweight), we rebalance it by "split"
- We split a node v to two new nodes u,u'
- starting from the bottom and going up



# Inserting and Deleting Keys in a B-tree

## Inserting Key x

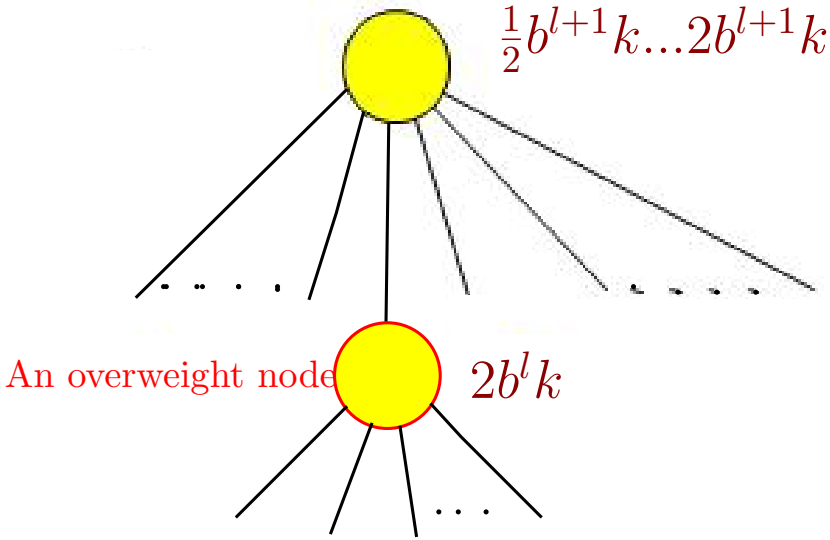
- Search for the key x, find node v that is parent of x
- Insert the key x to node v
- If at level l, w(v)=$2b^l k$ (overweight), we rebalance it by "split"
- We split a node v to two new nodes u,u'
- starting from the bottom and going up



# Inserting and Deleting Keys in a B-tree

## Inserting Key x

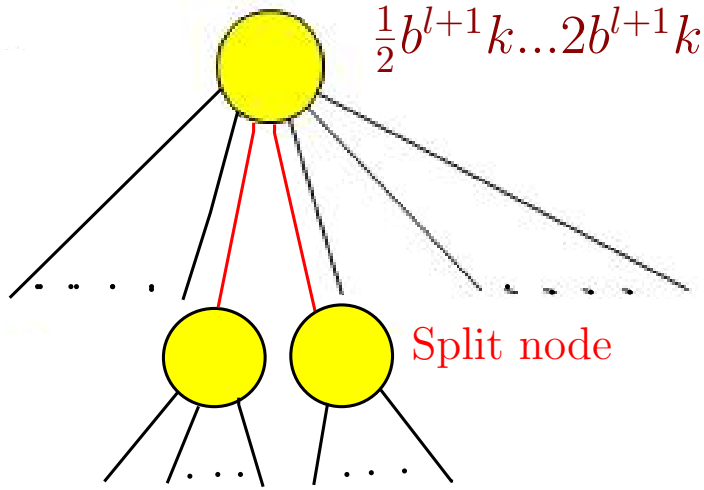
- Search for the key x, find node v that is parent of x
- Insert the key x to node v
- If at level l, w(v)=$2b^l k$ (overweight), we rebalance it by "split"
- We split a node v to two new nodes u,u'
- starting from the bottom and going up



# Inserting and Deleting Keys in a B-tree

## Inserting Key x

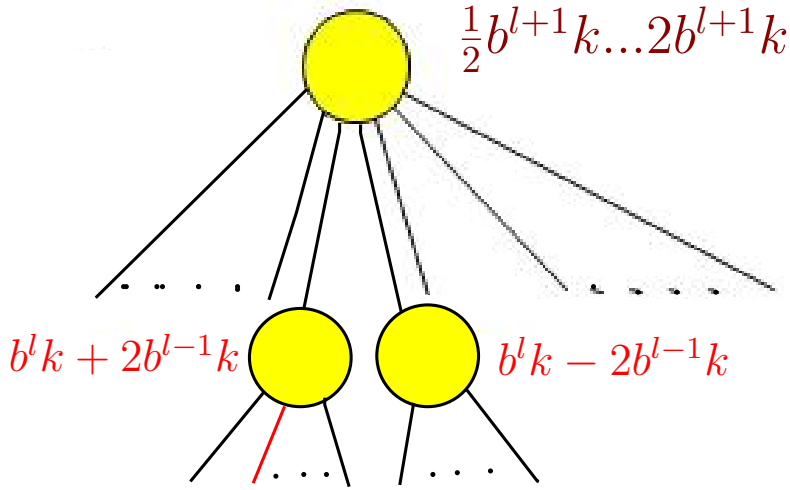
- Search for the key x, find node v that is parent of x
- Insert the key x to node v
- If at level l, w(v)=$2b^l k$ (overweight), we rebalance it by "split"
- We split a node v to two new nodes u,u'
- starting from the bottom and going up



# Inserting and Deleting Keys in a B-tree

## Inserting Key x

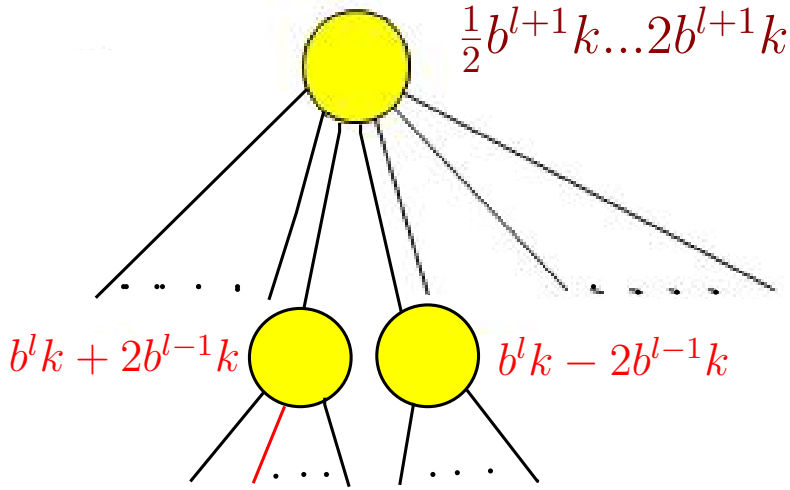
- Search for the key x, find node v that is parent of x
- Insert the key x to node v
- If at level i, w(v)=$2b^lk$ (overweight), we rebalance it by "split"
- We split a node v to two new nodes u,u'
- starting from the bottom and going up



# Inserting and Deleting Keys in a B-tree

## Inserting Key x

- Search for the key x, find node v that is parent of x
- Insert the key x to node v
- If at level i, w(v)=$2b^lk$ (overweight), we rebalance it by "split"
- We split a node v to two new nodes u,u'
- starting from the bottom and going up



## Inserting key x(continue)

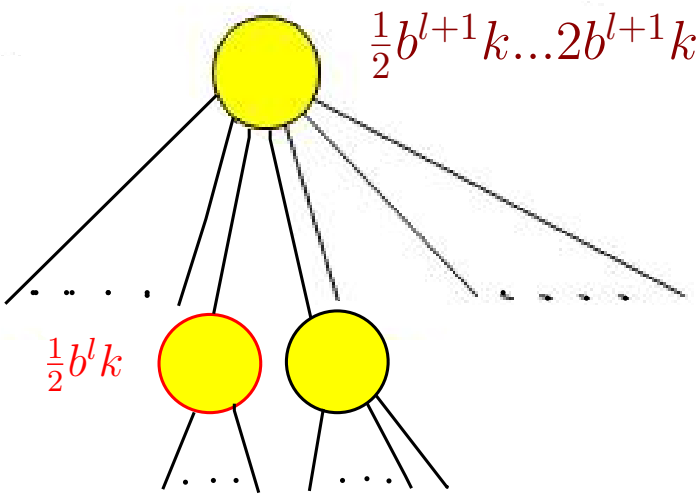
- $b^l k - 2b^{l-1}k \langle$ w(u),w(u') $\langle b^l k + 2b^{l-1}k$
- Since $b \geq 4$
- $\frac{1}{2}b^l k \langle$ w(u),w(u') $\langle \frac{3}{2}b^l k$
- The weight of each of these new nodes(u,u') is $\Omega(b^l)$

# Inserting and Deleting Keys in a B-tree(continue)

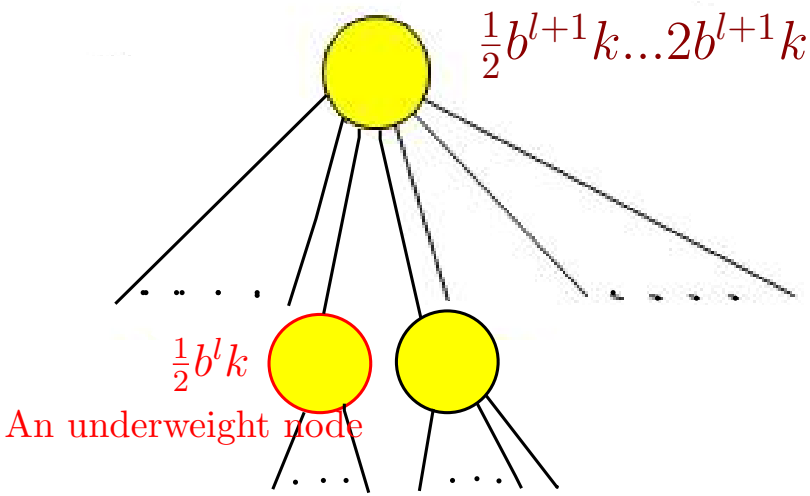
## Deleting Key x (fuse)

- Search for the key x to find the internal node v that is parent x
- Delete the key x from node v
- If at level l, w(v)$=\frac{1}{2}b^l k$ (underweight), we will rebalance it by "fuse" or "share" operations
- starting from the bottom and going up
- Node w:one of its nearest sibling of node v
- If w(w)$\leq \frac{5}{4}b^i k$ we do "fuse" operation

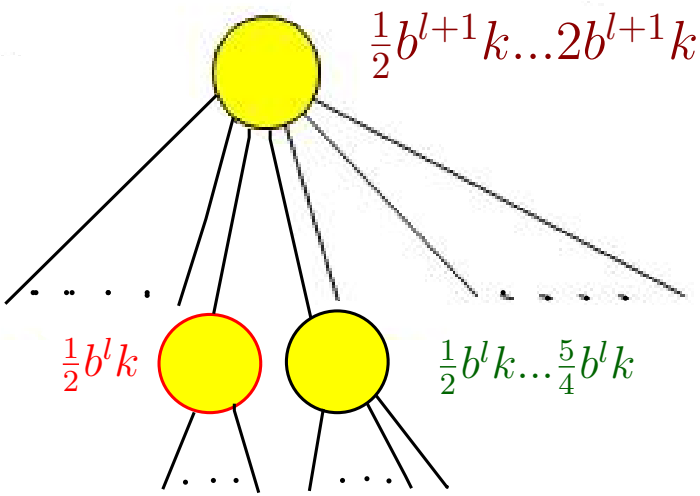
# Deleting Keys in a B-tree (fuse)



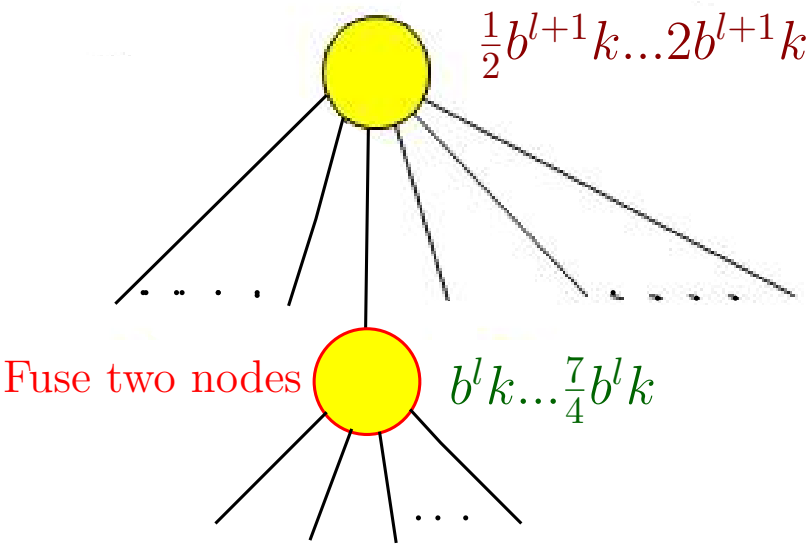
# Deleting Keys in a B-tree (fuse)



# Deleting Keys in a B-tree (fuse)

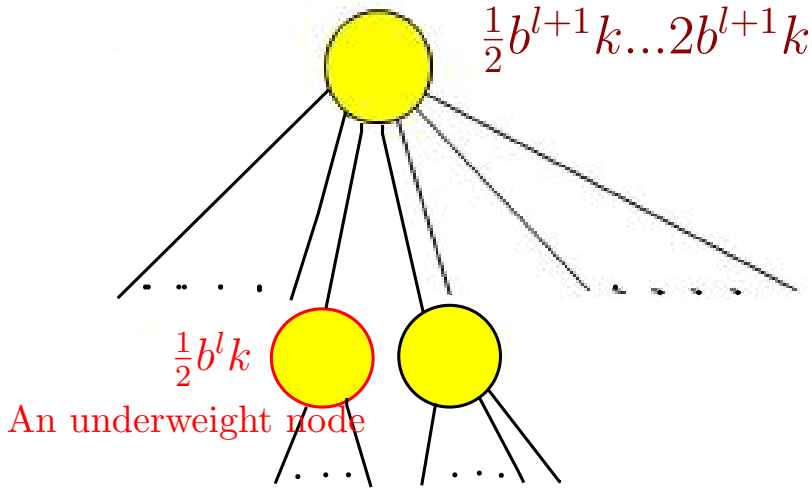


# Deleting Keys in a B-tree (fuse)



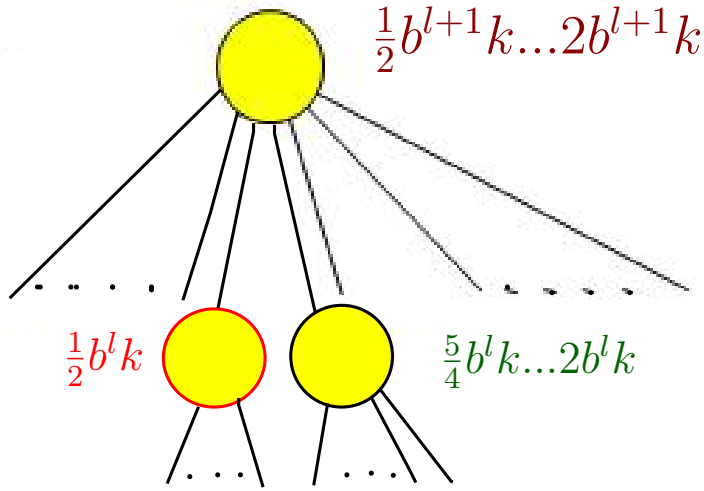
## Deleting Keys in a B-tree (share)

- if $\frac{5}{4}b^l k\langle$ w(w) $\langle 2b^l k$ we do "share" operation
- We have two new nodes u,u' result of "share"
- w(u)=$\frac{7}{8}b^l k - 2b^{l-1}k$
  w(u')=$\frac{5}{4}b^l k + 2b^{l-1}k$
- The weight of each of them(u,u') is $\Omega(b^l)$



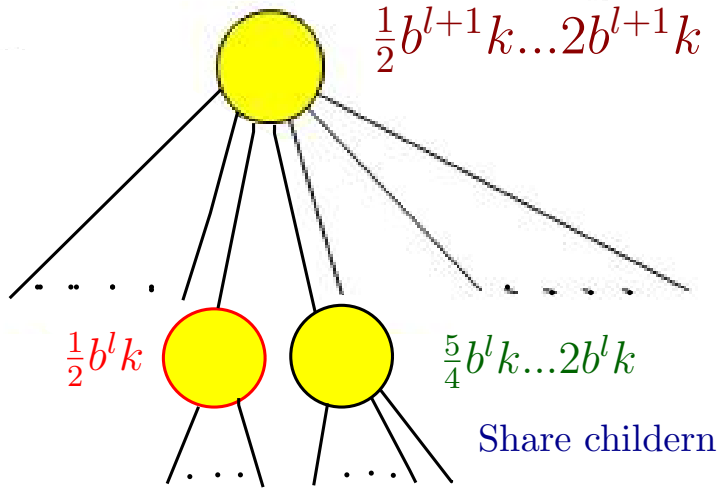
## Deleting Keys in a B-tree (share)

- if $\frac{5}{4}b^l k\langle$ w(w) $\langle 2b^l k$ we do "share" operation
- We have two new nodes u,u' result of "share"
- w(u)=$\frac{7}{8}b^l k - 2b^{l-1}k$
  w(u')=$\frac{5}{4}b^l k + 2b^{l-1}k$
- The weight of each of them(u,u') is $\Omega(b^l)$



## Deleting Keys in a B-tree (share)

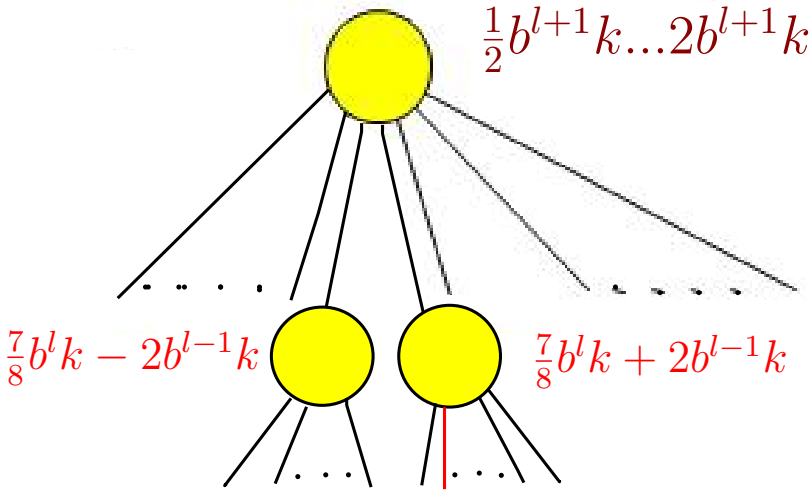
- if $\frac{5}{4}b^l k\langle$ w(w) $\langle 2b^l k$ we do "share" operation
- We have two new nodes u,u' result of "share"
- w(u)=$\frac{7}{8}b^l k - 2b^{l-1}k$
  w(u')=$\frac{5}{4}b^l k + 2b^{l-1}k$
- The weight of each of them(u,u') is $\Omega(b^l)$




Share childern of two nodes

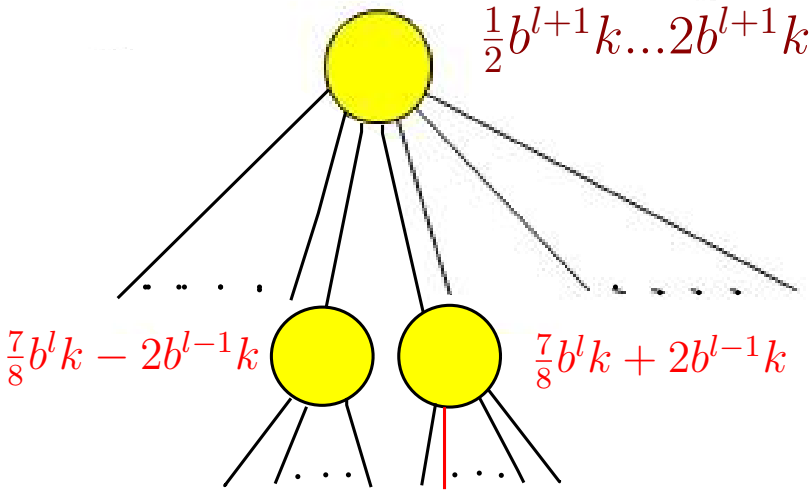
## Deleting Keys in a B-tree (share)

- if $\frac{5}{4}b^l k\langle$ w(w) $\langle 2b^l k$ we do "share" operation
- We have two new nodes u,u' result of "share"
- w(u)=$\frac{7}{8}b^l k - 2b^{l-1}k$
  w(u')=$\frac{5}{4}b^l k + 2b^{l-1}k$
- The weight of each of them(u,u') is $\Omega(b^l)$



## Deleting Keys in a B-tree (share)

- if $\frac{5}{4}b^l k\langle$ w(w) $\langle 2b^l k$ we do "share" operation
- We have two new nodes u,u' result of "share"
- w(u)=$\frac{7}{8}b^l k - 2b^{l-1}k$
  w(u')=$\frac{5}{4}b^l k + 2b^{l-1}k$
- The weight of each of them(u,u') is $\Omega(b^l)$



# Analysis of inserting and deleting in B-tree

- The cost of rebalancing a node: O(1) I/Os
- The total cost of B-tree rebalancing: $O(log_b^N)$ I/Os
- We have in fact shown something stronger
- The weight of node v at level i, $W = \Theta(b^i)$
- To assume $S$ : an auxiliary data structure used when searching in the v's subtree
- When v is rebalanced we spend f(W) I/Os to compute $S$

# Analysis(continue)

- The rebalancing operation have $\Omega(W)$ insertions and deletions in v's subtree and also in $S$
- The amortized cost of maintaining $S$ :O($f(W)/W$) I/Os per node on the search path of an update
  or O($\frac{f(W)}{W} log_b^N$) I/Os per update
- As an example,if f(W)=O(W/B) I/Os
- The amortized cost per update is O( $\frac{1}{B} log_b^N$) I/Os
- that this is negligible

# B-tree Variants

# B-tree Variants

## 1.Parent Pointers and Level Links

- Maintain a pointer to the parent of each node
- Maintain all nodes at each level with a doubly linked list
- One application of these pointers is a "finger search"
- Given a leaf v in the B-tree, search for another leaf w
- Q: the number of leaves between v and w
- The number of I/Os: $O(log_b^Q)$

# B-tree Variants

## 1.Parent Pointers and Level Links

- Maintain a pointer to the parent of each node
- Maintain all nodes at each level with a doubly linked list
- One application of these pointers is a "finger search"
- Given a leaf v in the B-tree, search for another leaf w
- Q: the number of leaves between v and w
- The number of I/Os: $O(log_b^Q)$

## 2.String B-trees

- We have assumed that the B-tree's keys have fixed length
- In some applications the keys are strings of unbounded length
- all the usual B-tree operations,can be efficiently supported in this setting

# B-tree Variants

## 3.Divide and Merge Operations

- We have two useful applications
- Divide a B-tree into two parts
- Merge two B-trees "glue"
- These operations can be supported in $O(log_b^N)$ I/Os

# Batched Dynamic Problems

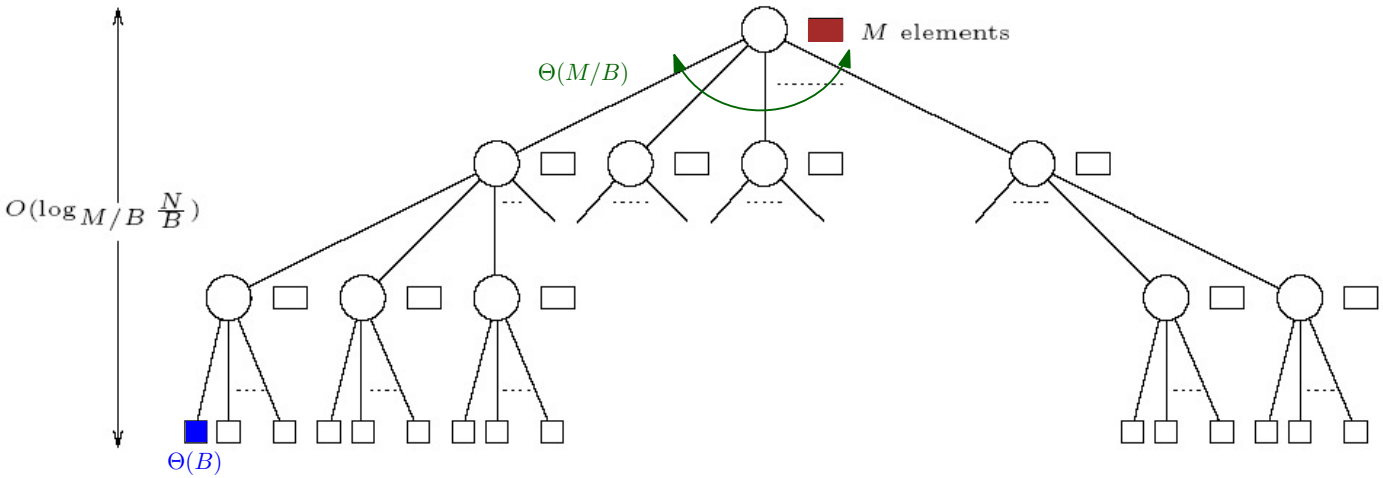
- B-trees answer queries in an on-line fashion
- In batched dynamic problems a *batch* of updates and queries is provided to the data structure
- Only at the end of the batch, the data structure delivers the answers

**The batched range searching**

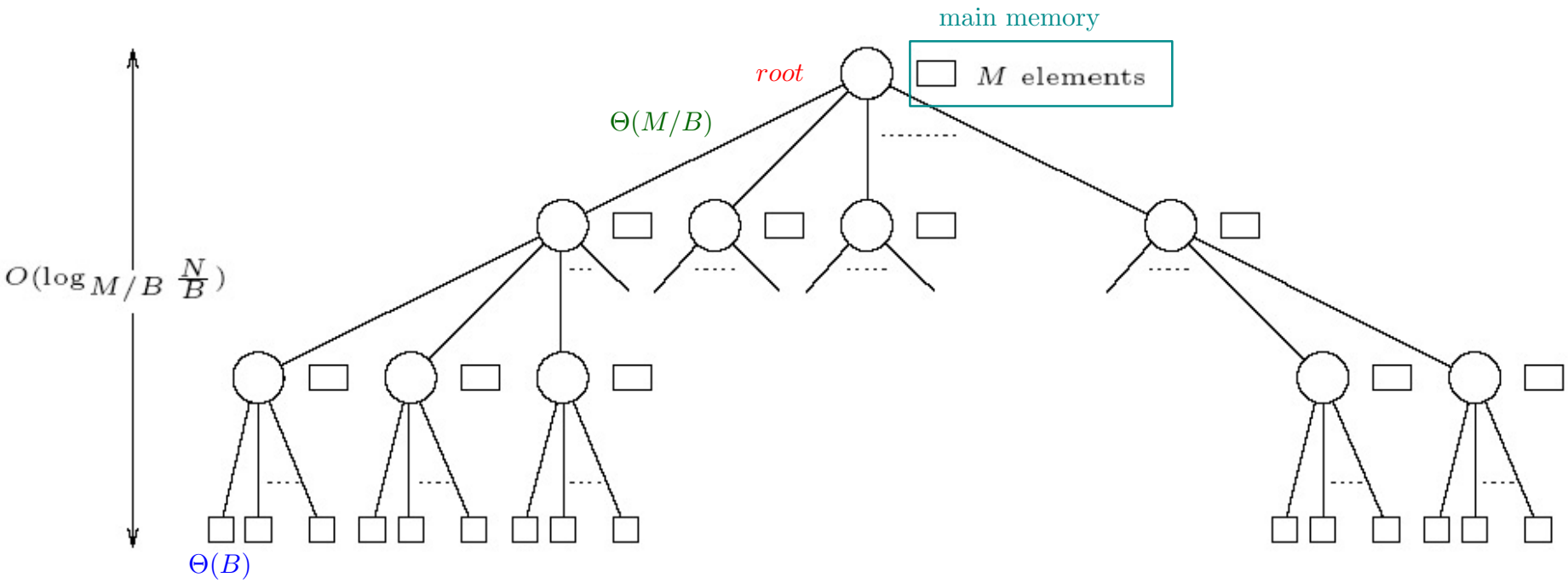
- Given a sequence of insertions and deletions of integers
- Each query of integers is compared with the sequense and reported

# Buffer trees

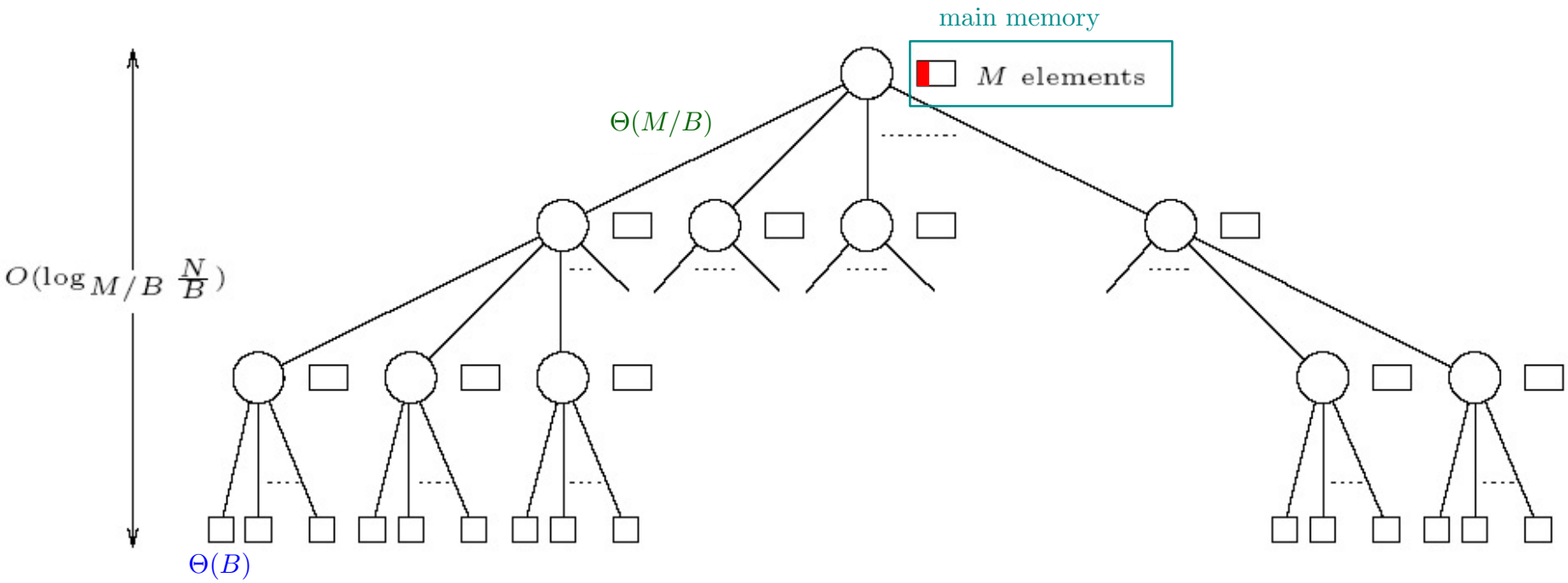
- The buffer tree technique has been used for I/O optimal algorithms
- Each internal node has an buffer with size $\Theta(M)$
- A buffer tree has degree $\Theta(M/B)$
- Leaves contain $\Theta(B)$ keys
- Root buffer reside entirely on main memory
- Non-root buffers reside entirely on external memory



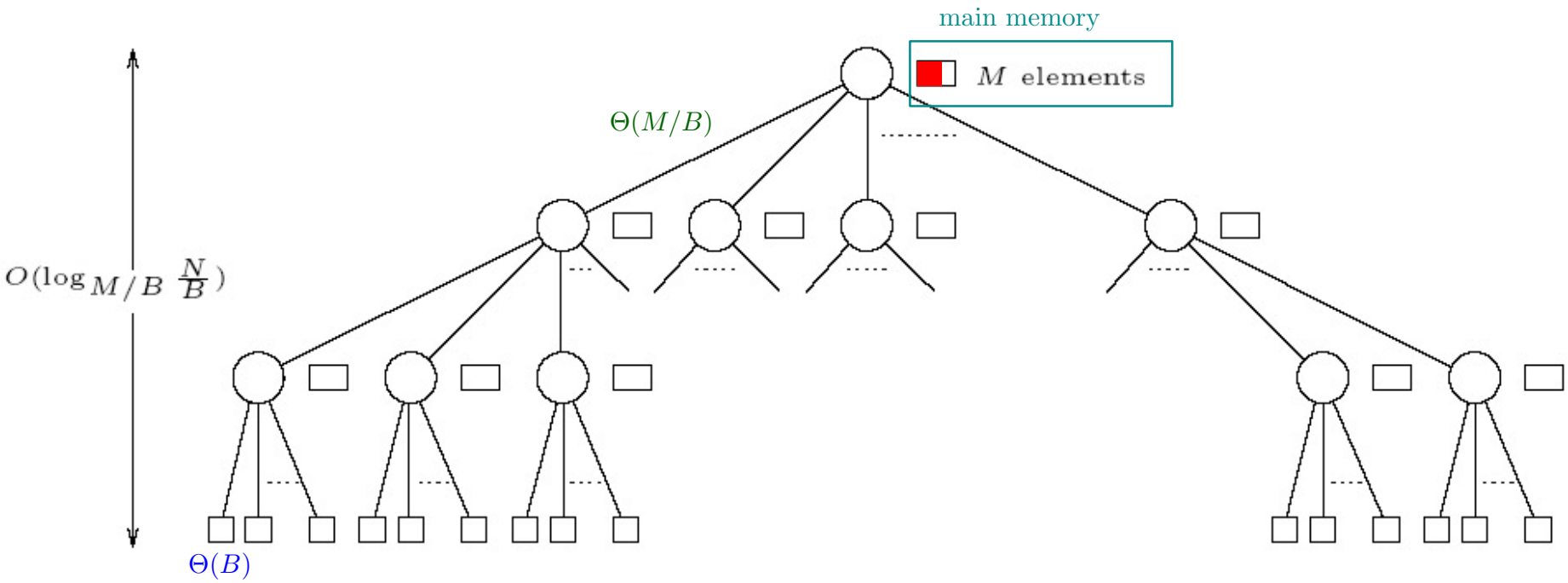
# How does a buffer tree work?



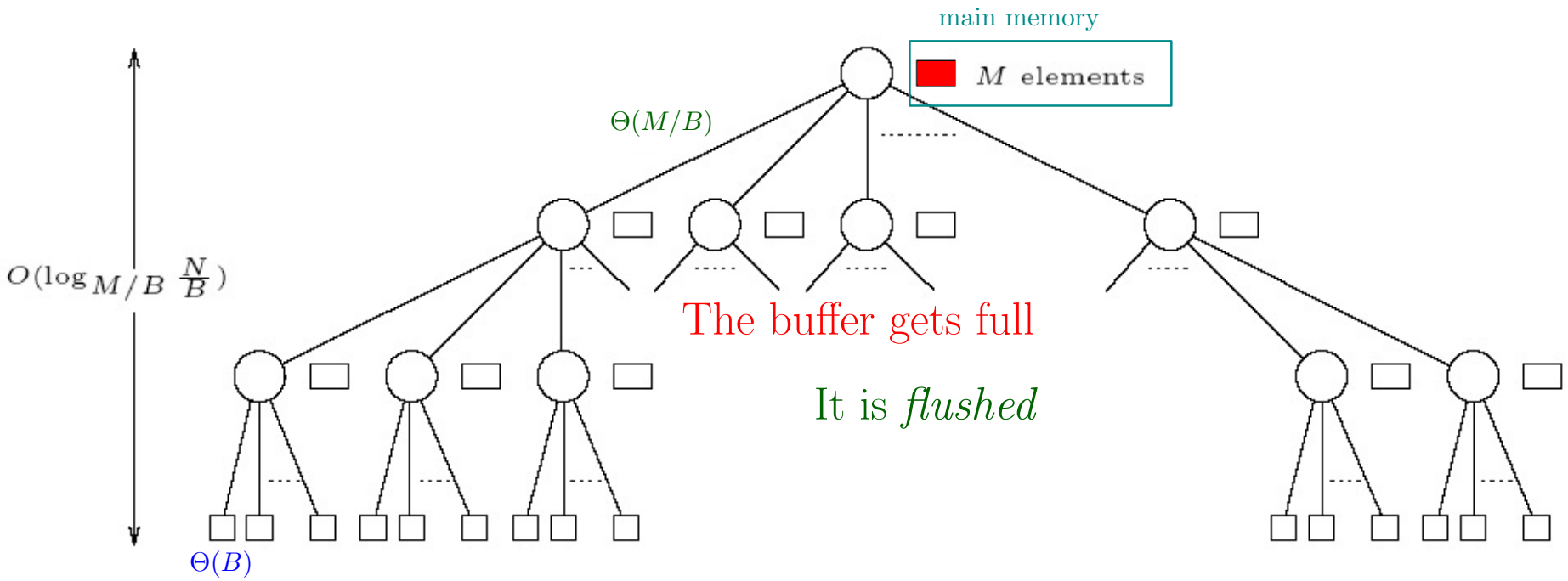
# How does a buffer tree work?



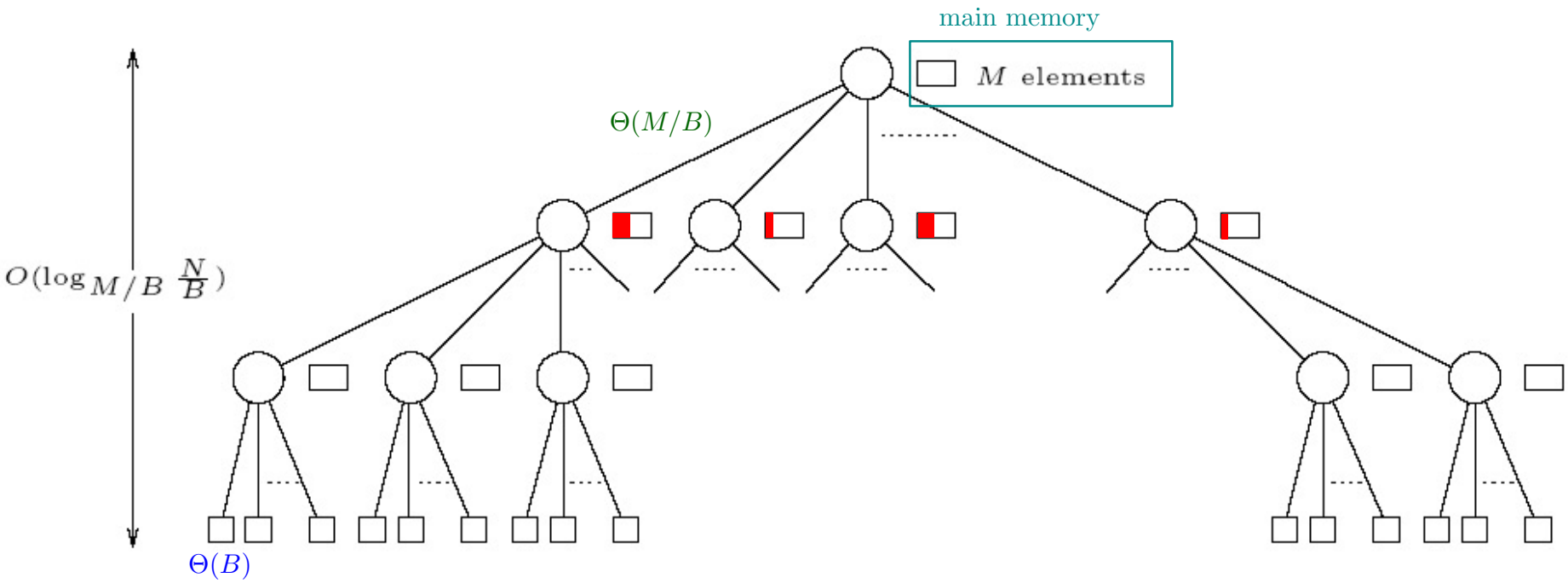
# How does a buffer tree work?



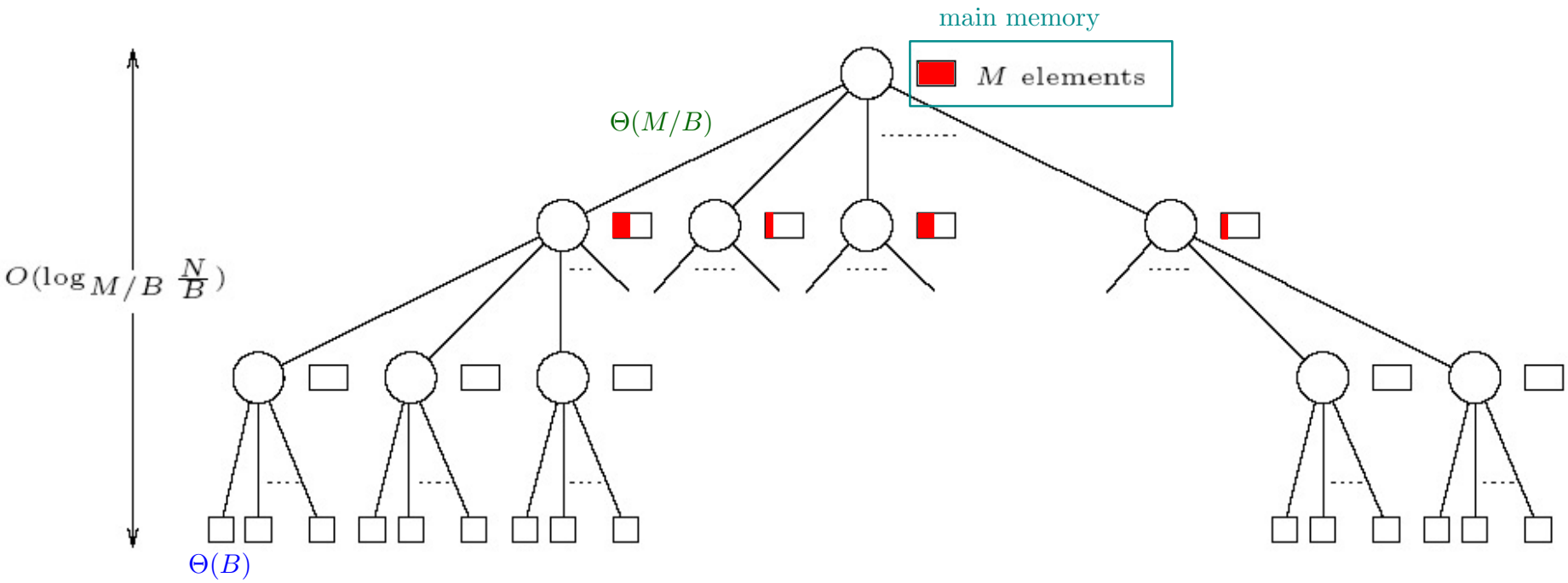
# How does a buffer tree work?



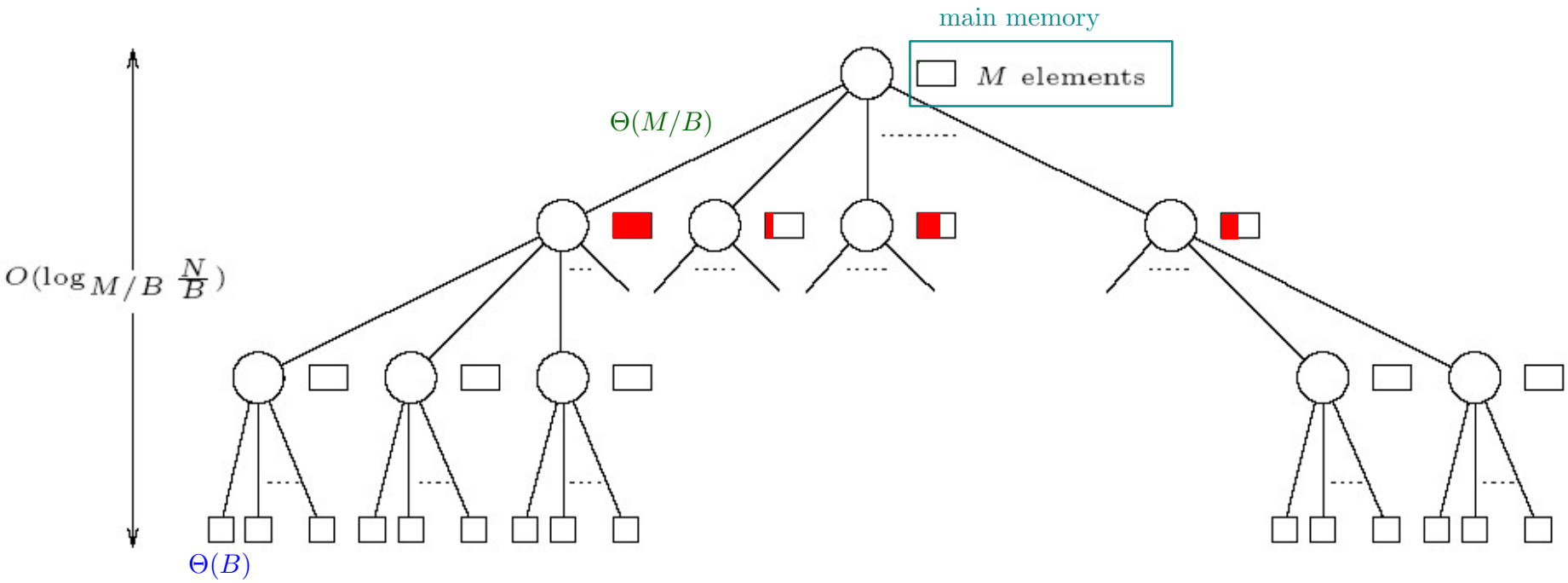
# How does a buffer tree work?



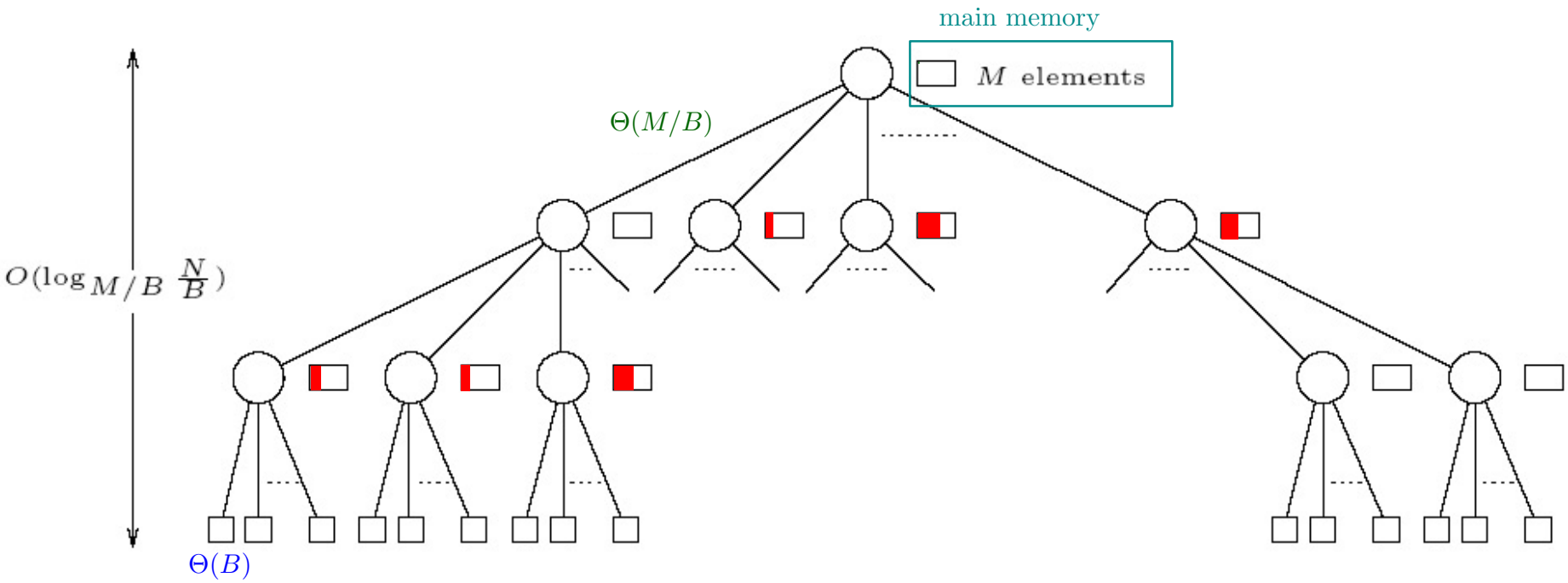
# How does a buffer tree work?



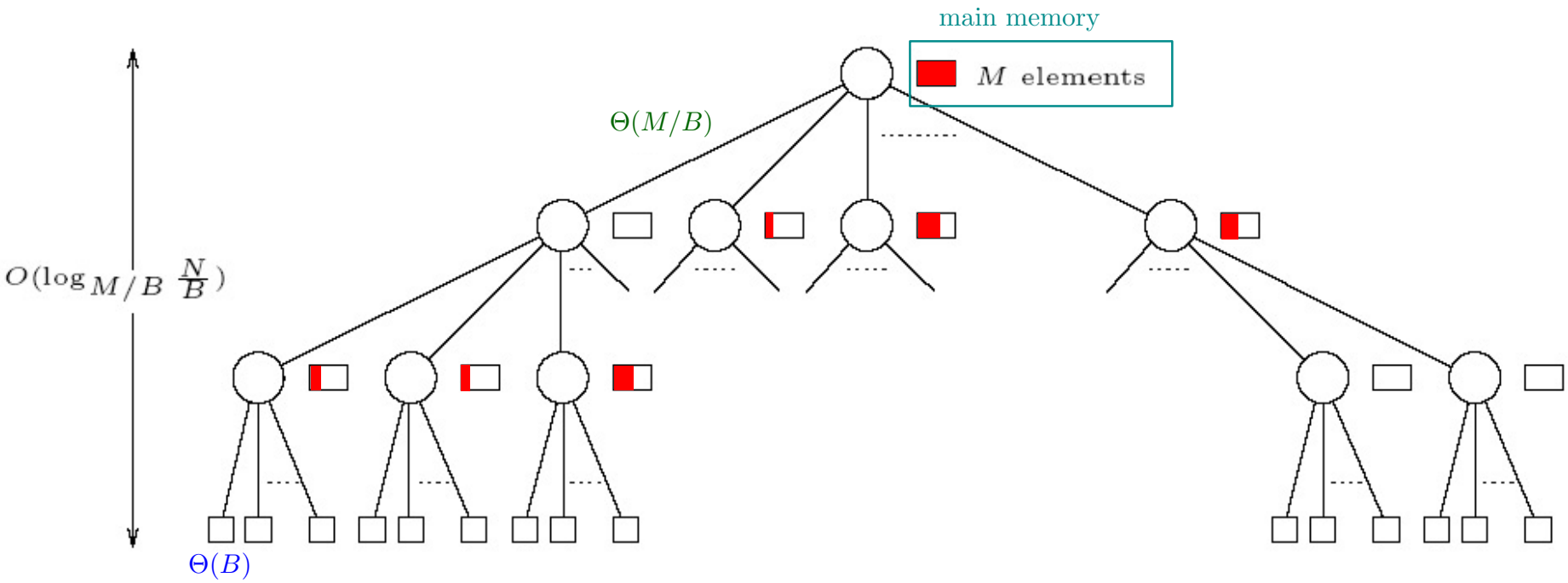
# How does a buffer tree work?



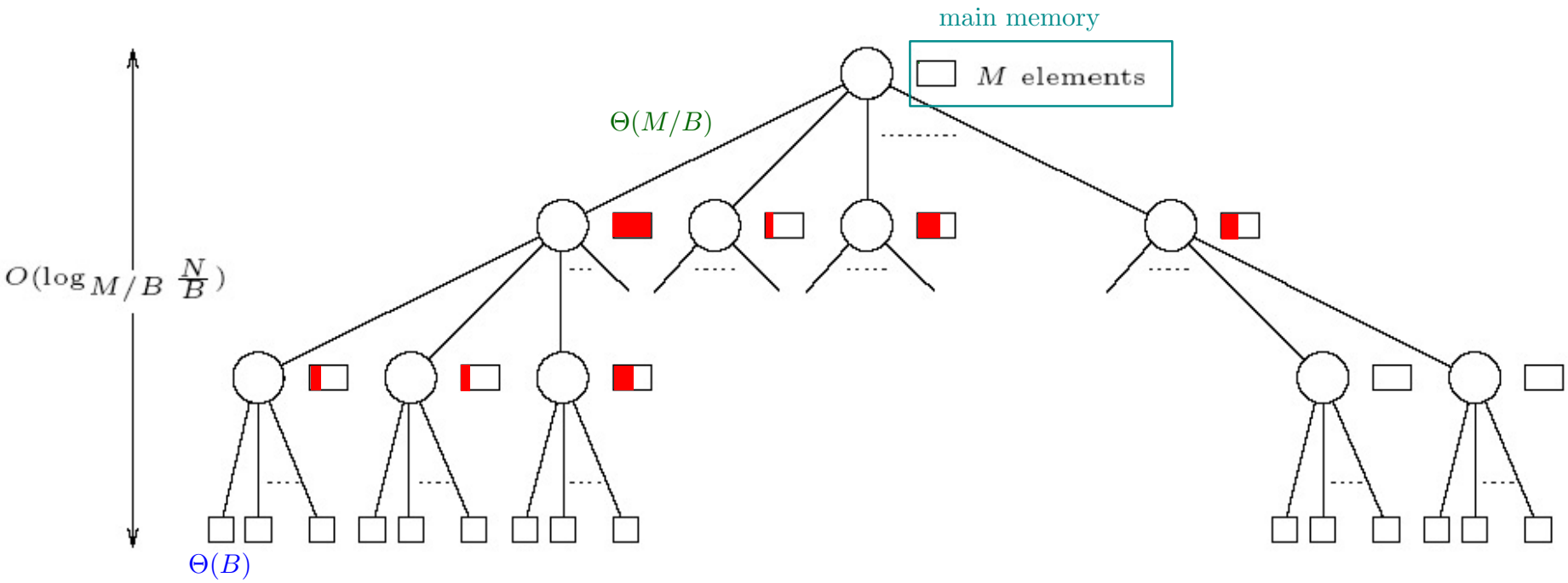
# How does a buffer tree work?



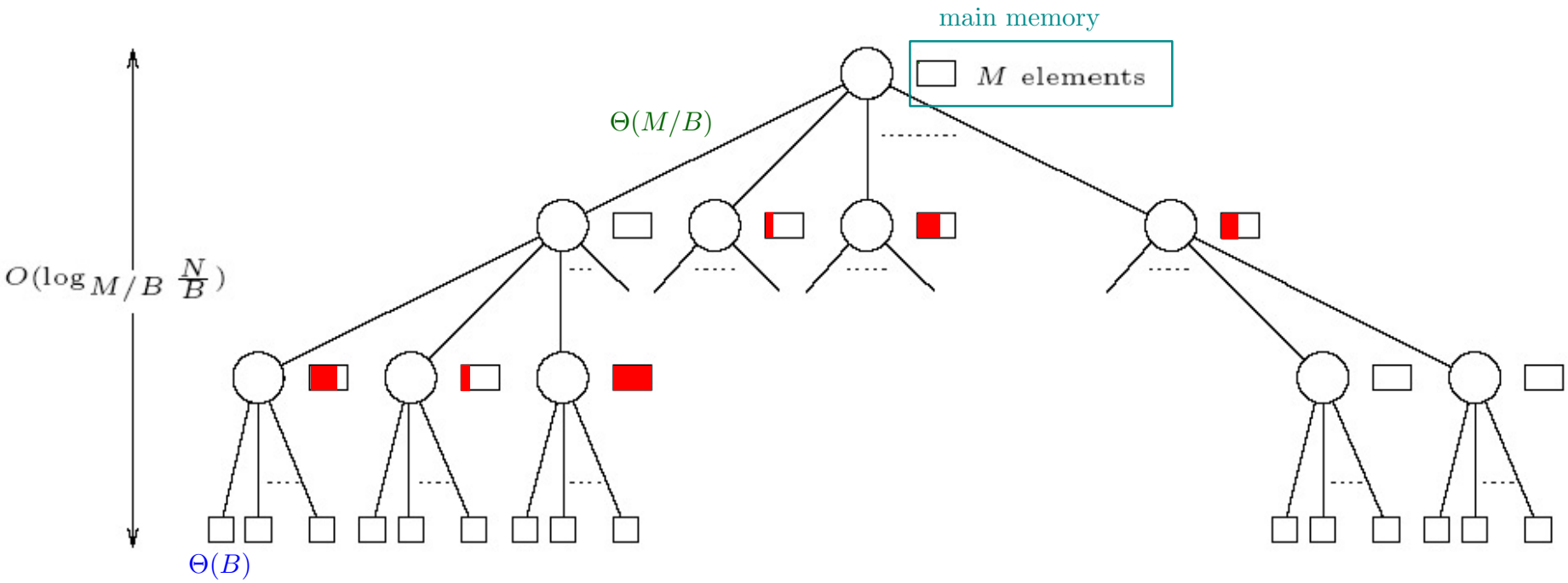
# How does a buffer tree work?



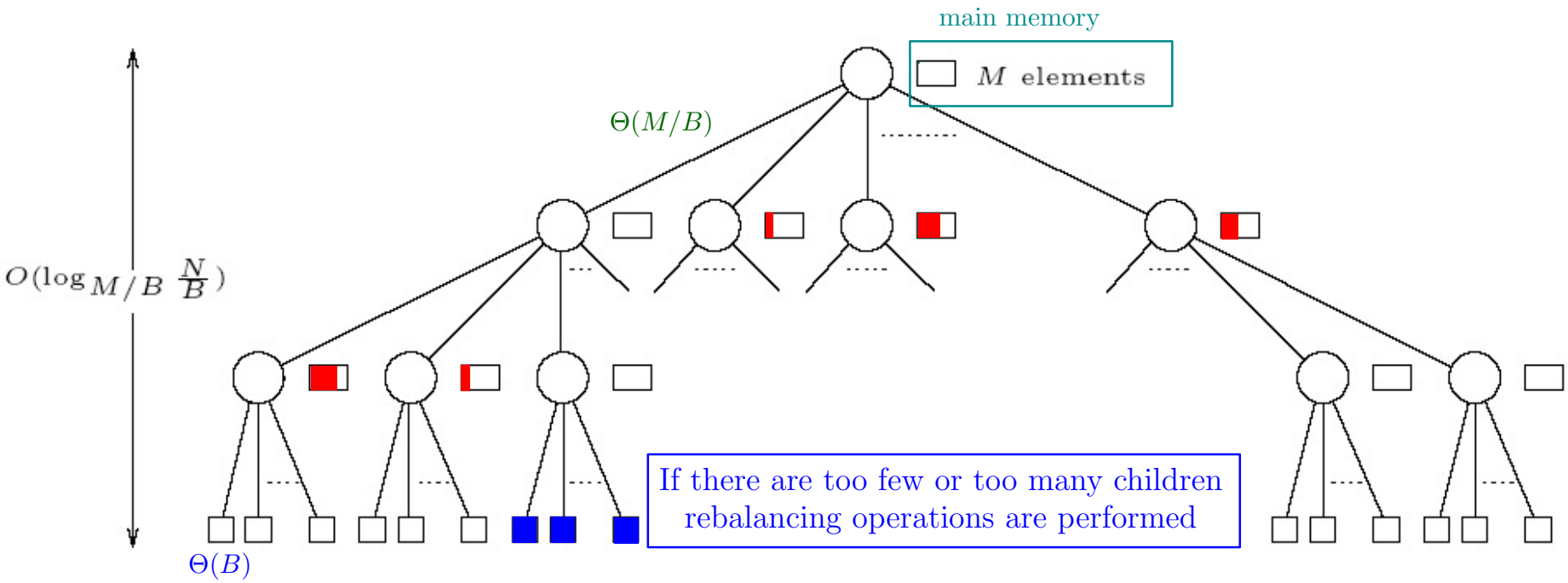
# How does a buffer tree work?



# How does a buffer tree work?



# How does a buffer tree work?



# I/O Analysis for Buffer tree

# I/O Analysis for Buffer tree

## The cost of flushing a buffer

- O(M/B) I/Os for reading the buffer
- O(M/B) I/Os for writing the operations to the buffers of the children

# I/O Analysis for Buffer tree

## The cost of flushing a buffer

- O(M/B) I/Os for reading the buffer
- O(M/B) I/Os for writing the operations to the buffers of the children

## The cost of all of flushes $O(\frac{1}{B} log_{\frac{M}{B}}^{\frac{N}{B}})$ I/Os per operation

- A flushing costs O(1/B) I/Os per operation in the buffer

# I/O Analysis for Buffer tree

## The cost of flushing a buffer

- O(M/B) I/Os for reading the buffer
- O(M/B) I/Os for writing the operations to the buffers of the children

## The cost of all of flushes $O(\frac{1}{B} log_{\frac{M}{B}}^{\frac{N}{B}})$ I/Os per operation

- A flushing costs O(1/B) I/Os per operation in the buffer

## The total cost of rebalancing during N updates is O(N/B) I/Os

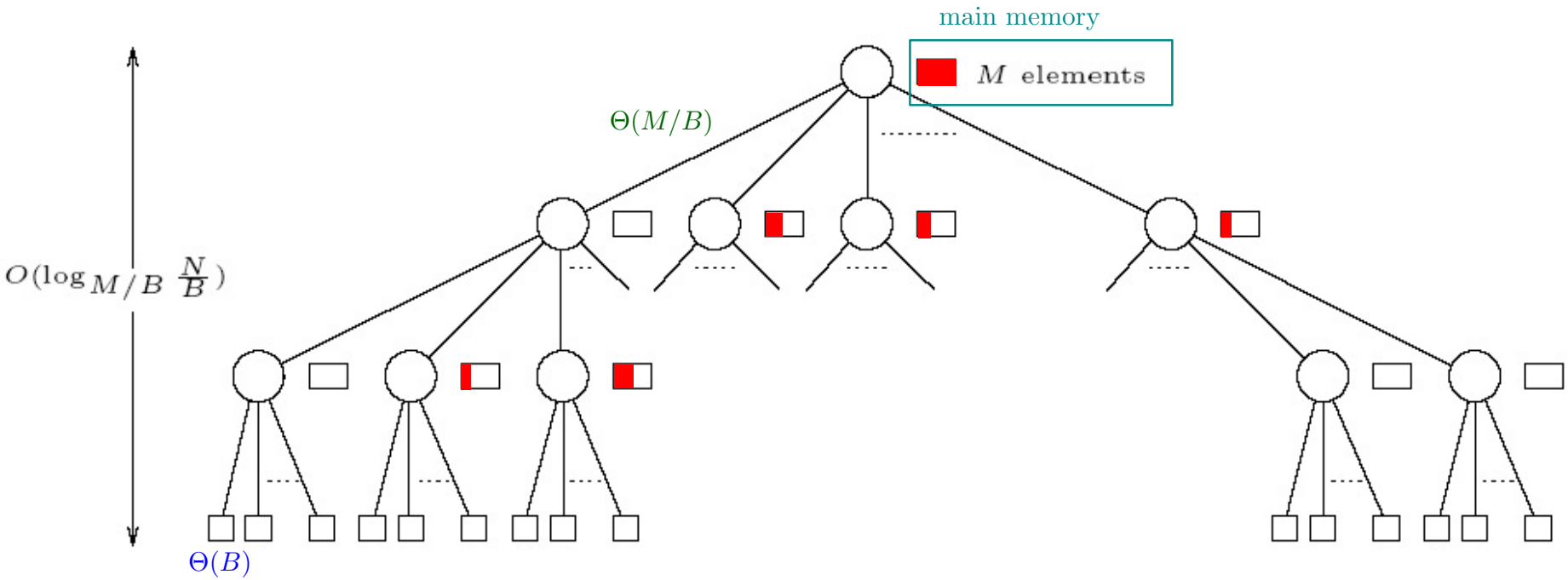
- The cost of a rebalancing operation on a node is O(M/B) I/Os
- Number of nodes that need to rebalancing operations during N updates is O(N/M)

# Priority Queues

- The basic operations insertion of a key, finding the smallest key, and deleting the smallest key
- Sometimes additional operations are supported, such as deleting an arbitrary key and decreasing the value of a key
- we use buffering technique for priority queue
- The entire buffer of the root node and the O(M/B) leftmost leaves are always kept in internal memory

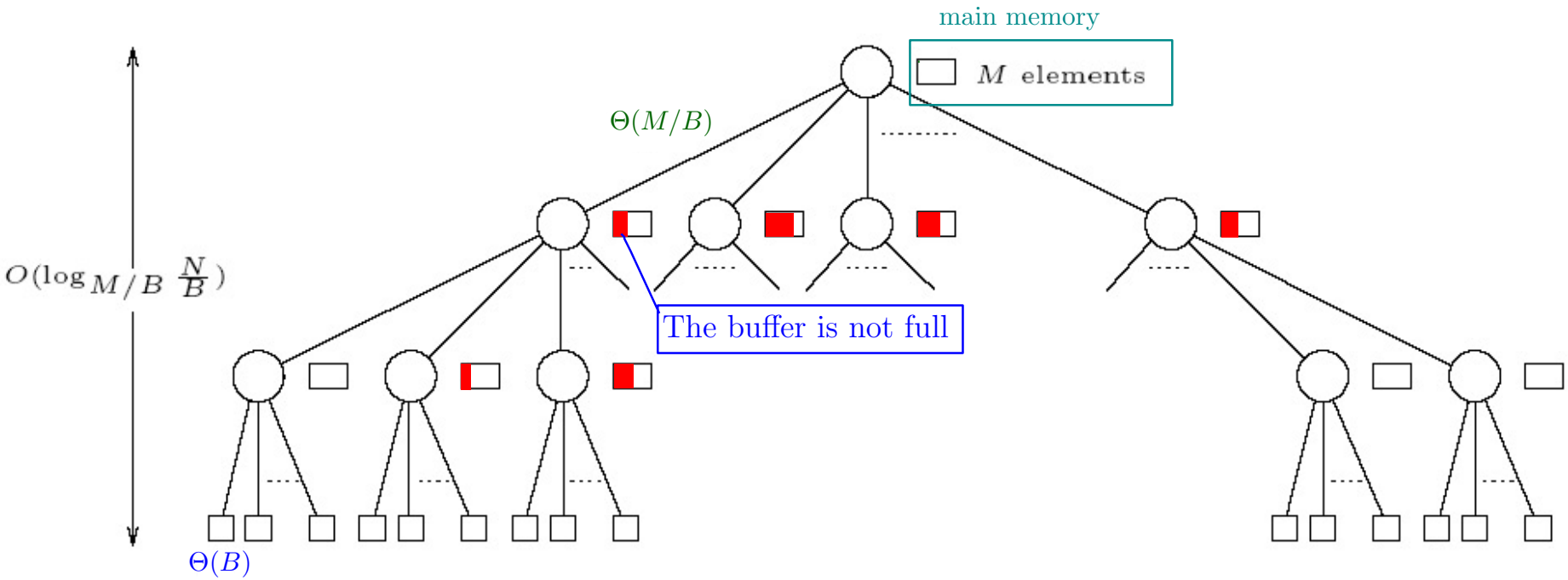
## How does priority queue using buffer tree work?

- All buffers on the path from the root to the leftmost leaf must be empty
- For this,Whenever the root is flushed we also flush all buffers down the leftmost path



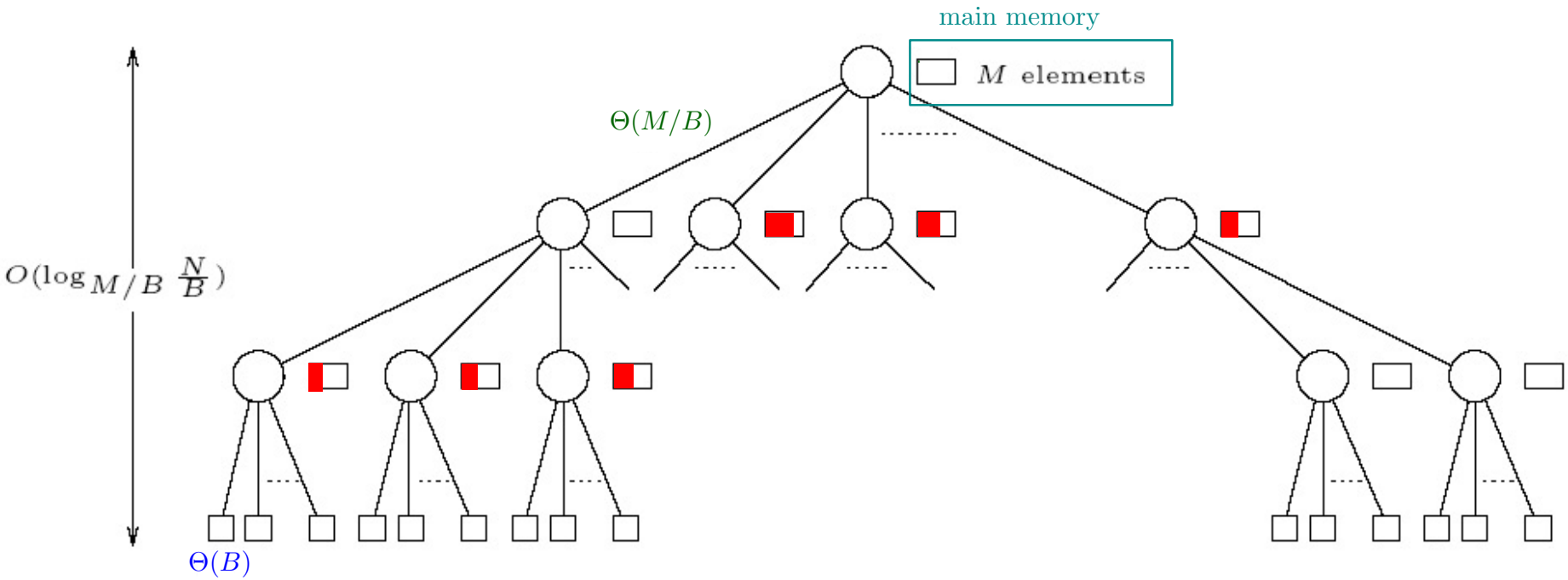
## How does priority queue using buffer tree work?

- All buffers on the path from the root to the leftmost leaf must be empty
- For this,Whenever the root is flushed we also flush all buffers down the leftmost path



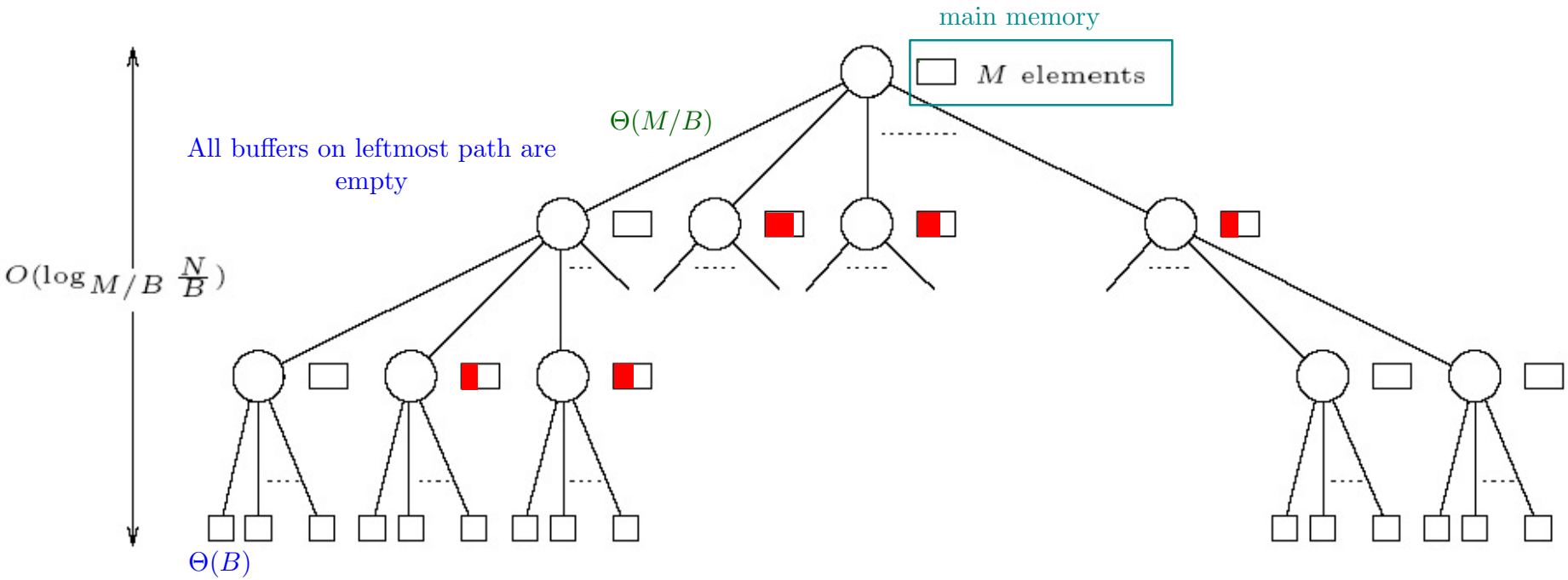
## How does priority queue using buffer tree work?

- All buffers on the path from the root to the leftmost leaf must be empty
- For this,Whenever the root is flushed we also flush all buffers down the leftmost path



## How does priority queue using buffer tree work?

- All buffers on the path from the root to the leftmost leaf must be empty
- For this,Whenever the root is flushed we also flush all buffers down the leftmost path



## I/O Analysis for Priority Queues

- All buffers on the leftmost path are flushed with $O(\frac{M}{B} log \frac{\frac{N}{B}}{\frac{M}{B}})$ I/Os
- We have O(M) operations with each flush of the root buffer
- The amortized cost of these extra flushes is $O(\frac{1}{B} log \frac{\frac{N}{B}}{\frac{M}{B}})$ I/Os per operation

## I/O Analysis for Priority Queues

- All buffers on the leftmost path are flushed with $O(\frac{M}{B} log \frac{\frac{N}{B}}{\frac{M}{B}})$ I/Os
- We have O(M) operations with each flush of the root buffer
- The amortized cost of these extra flushes is $O(\frac{1}{B} log \frac{\frac{N}{B}}{\frac{M}{B}})$ I/Os per operation

## Results

- Find-minimum queries can be answered on-line without using any I/Os
- It can shown that is impossible to perform insertion and delete minimums in $o(\frac{1}{B} log \frac{\frac{N}{B}}{\frac{M}{B}})$ I/Os
- Open problems

- Hashing Based Dictionaries

# Lookup with Good Expected Performance

- We will consider *linear probing* and *chaining with separate lists*
- These schemes need only a single hash function h in internal memory
- We assume that any hash function value h(x) is uniformly random

# Lookup with Good Expected Performance

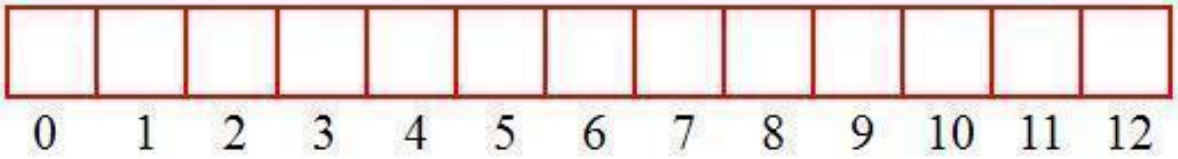
- We will consider *linear probing* and *chaining with separate lists*
- These schemes need only a single hash function h in internal memory
- We assume that any hash function value h(x) is uniformly random

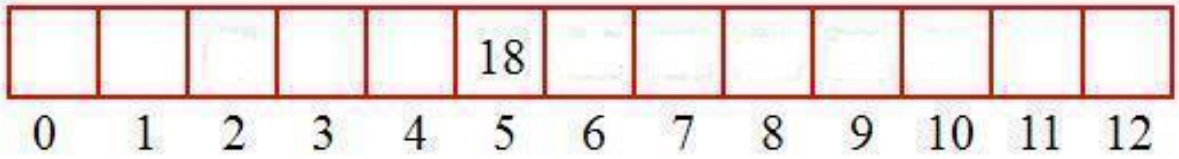
## Load factor $\alpha$

- M is the number of different addresses are produced by hash function and N is the number of keys
- $\alpha = \frac{N}{M}$

# Linear Probing Example

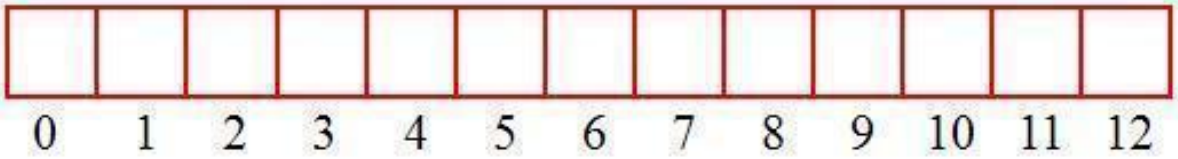
- h(k) = $k$ mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73

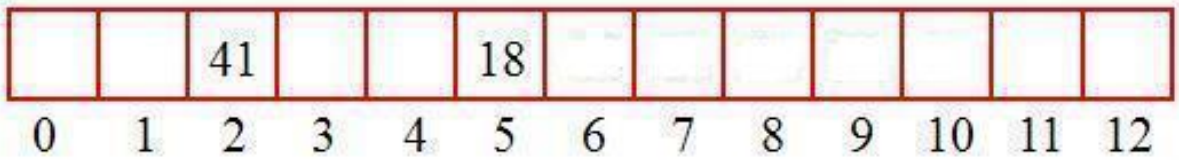


# Linear Probing Example

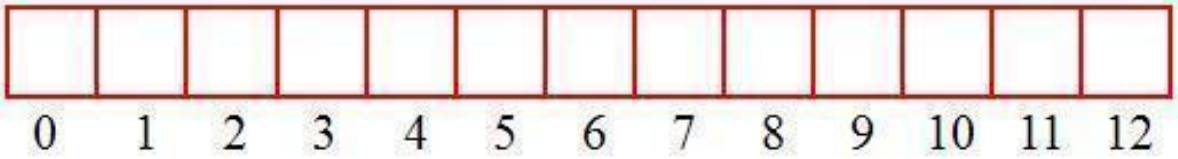
- h(k) = $k$ mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73

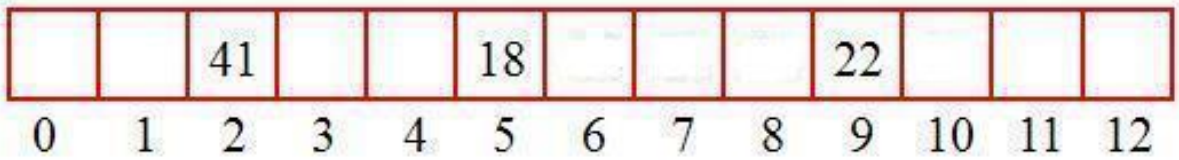


# Linear Probing Example

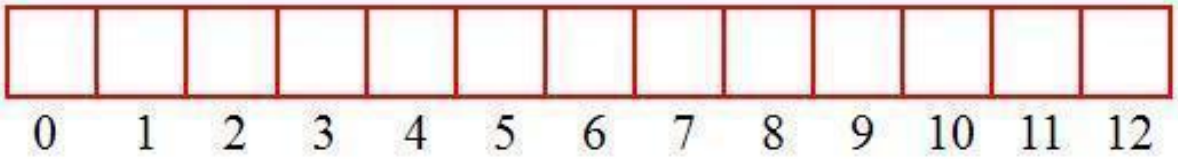
- h(k) = $k$ mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73



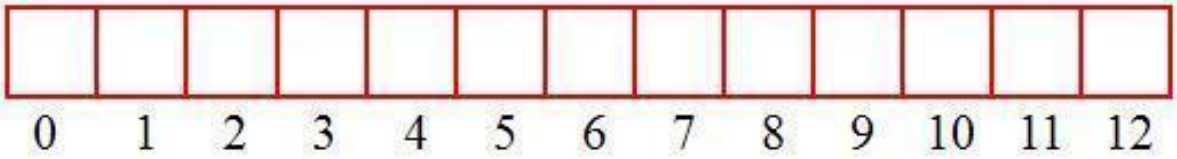
# Linear Probing Example

- h(k) = $k$ mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73



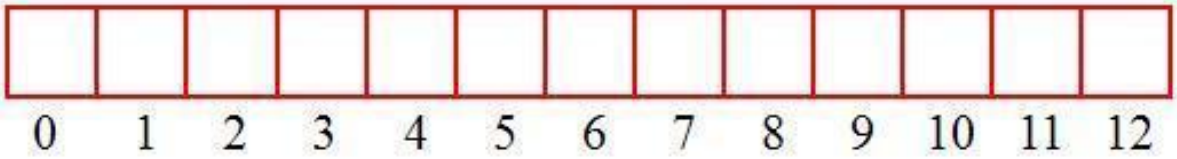

# Linear Probing Example

- h(k) = $k$ mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73



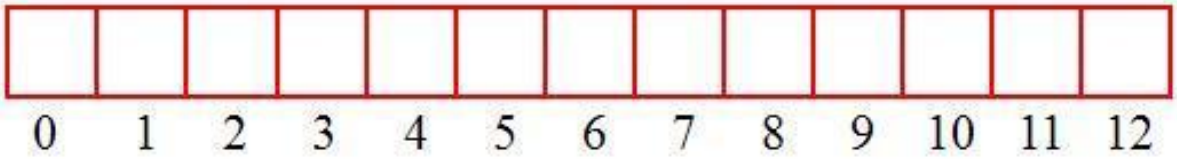

# Linear Probing Example

- h(k) = $k$ mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73



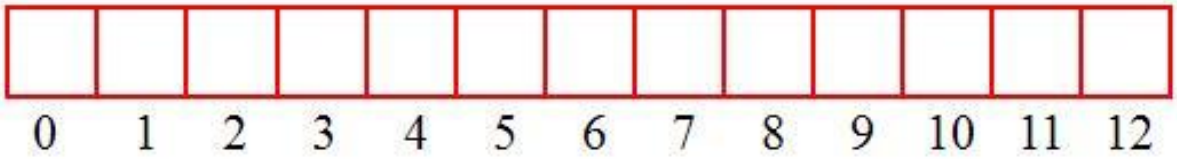

# Linear Probing Example

- h(k) = $k$ mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73




# Linear Probing Example

- h(k) = $k$ mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73




# 1.Linear Probing

## Operations

# 1.Linear Probing

## Operations

- Insertion

# 1.Linear Probing

## Operations

- Insertion
- Deletion

# 1.Linear Probing

## Operations

- Insertion
- Deletion
- Lookup

# 1.Linear Probing

## Operations

- Insertion
- Deletion
- Lookup

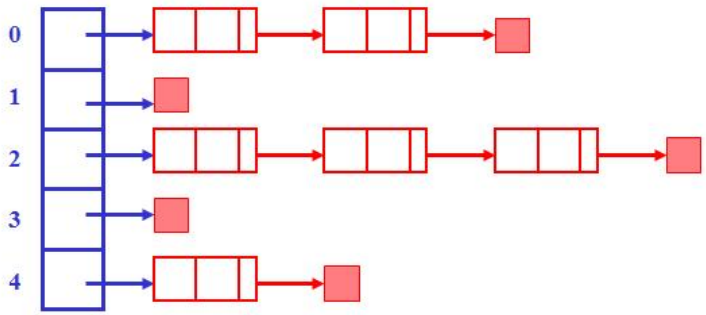
## The Number of I/Os for a Lookup

- The expected average number of I/Os for a lookup is $1 + (1 - \alpha)^{-2}2^{-\Omega(B)}$
- $\alpha \leqslant 1 - \varepsilon$ and B is not too small $\Longrightarrow$ the expected average is very close to 1
- The probability of using k (more than one) I/Os for a lookup is $2^{-\Omega(B(k-1))}$

# 2.Chaining with Separate Lists

## Chaining works faster than Linear Probing

- Each block in the hash table is the start of a linked list of keys hashing to that block
- When the pseudo random function works truly, all lists will consist of just a single block
- The probability that more than kB keys hash to a certain block is at most $e^{-\alpha B(k/\alpha-1)^2/3}$ (Chernoff bounds)
- The probabilities decrease faster with k than in linear probing
- If B is large and the load factor is not too high, overflows will be very rare



# Lookup Using One External Memory Access

# Lookup Using One External Memory Access

## 1-Making Use of Internal Memory

If sufficient internal memory is available, searching in a dictionary can be done in a single I/O with two approaches:

1. Overflow area
2. Perfect hashing and extendible hashing

# Lookup Using One External Memory Access

## 1-Making Use of Internal Memory

If sufficient internal memory is available, searching in a dictionary can be done in a single I/O with two approaches:

1. Overflow area
2. Perfect hashing and extendible hashing

## 2-Using a Predecessor Dictionary

If we increase internal computation, both internal and external space usage can be made better than of extendible hashing

# Overflow area

## First Idea

- Internal memory for $2^{-\Omega(B)}N$ keys and associated information is available
- Store the keys that can not be accommodated externally in an internal memory dictionary
- The probability that be more than $2^{-c(\alpha)\Omega(B)}N$ such keys is so small
- If it happens we rehash, choose a new hash function to replace h

# Overflow area (continue)

## Second Idea

The overflow area can reside in external memory

For single I/O lookups, internal memory data structures must:

1. Identify blocks that have overflown
2. Facilitate single I/O lookup of the elements hashing to these blocks

# Overflow area (continue)

## Second Idea

The overflow area can reside in external memory

For single I/O lookups, internal memory data structures must:

1. Identify blocks that have overflown
2. Facilitate single I/O lookup of the elements hashing to these blocks

## First Task

- It be solved by maintaining a dictionary of overflowing blocks
- This requires $O(2^{-c(\alpha)B}NlogN)$ bits of internal space

# Overflow area (continue)

## Second Idea

The overflow area can reside in external memory

For single I/O lookups, internal memory data structures must:

1. Identify blocks that have overflown
2. Facilitate single I/O lookup of the elements hashing to these blocks

## First Task

- It be solved by maintaining a dictionary of overflowing blocks
- This requires $O(2^{-c(\alpha)B}NlogN)$ bits of internal space

## Second Task

- It be solved recursively by a dictionary supporting single I/O lookups
- Store a set that with high probability has size $O(2^{-c(\alpha)B}N)$

# Perfect hashing

## Mairson introduced a B-perfect hash function

- Hash function p : K $\longrightarrow \{1, ..., \lceil N/B \rceil\}$
- It maps at most B keys to each block
- A function uses O(Nlog(B)/B) bits of internal memory
- If the number of blocks is $\lceil N/B \rceil$, this is the best possible

# Perfect hashing

## Mairson introduced a B-perfect hash function

- Hash function p : K $\longrightarrow \{1, ..., \lceil N/B \rceil\}$
- It maps at most B keys to each block
- A function uses O(Nlog(B)/B) bits of internal memory
- If the number of blocks is $\lceil N/B \rceil$, this is the best possible

## Disadvantages

1. The time and space needed to evaluate this hash functions is extremely high
2. It seems very difficult to obtain a dynamic version

# Extendible Hashing

- Use an internal structure called a *directory*
- Directory is an array of $2^d$ pointers to external blocks
- Random hash function h : K $\longrightarrow \{0,1\}^r$ for $r \geqslant d$
- Lookup of a key k is performed by using $h(k)_d$
- $h(k)_d$ is d least significant bits of h(k) for determine an entry in the directory
- The parameter d is the smallest number that with it at most B dictionary keys map to the same value under $h(k)_d$
- If $r \geqslant 3logN$, such a d exists with high probability, else we rehash it

# Extendible Hashing(continue)

## The Main Results

- Lookups uses a single I/O and constant internal processing time
- The expected number of directory's entries is $4\frac{N}{B}N^{1/B}$
- If we have N/B blocks $\Rightarrow$ we require $\frac{1}{2}Nlog(B)/B + \Theta(N/B)$ bits of internal space (it is close to optimal)
- It can be shown that about 69 percent of the space is utilized

# Extendible Hashing(continue)

## Extendible Hashing adapts to changes of the key set

- The level of a block is the largest $d' \leqslant d$ for which all its keys map to the same value under $h_{d'}$
- Whenever a block at level $d'$ has run full,it is split into two blocks at level $d' + 1$ using $h_{d'+1}$
- In case $d' = d$ we first need to double the size of the directory
- If two blocks at level $d'$ with keys having the same function value under $h_{d'-1}$ contain less than B keys in total, these blocks are merged
- If no blocks are left at level d, the size of the directory is halved

# Lookup Using Two Parallel External Memory Accesses

# Lookup Using Two Parallel External Memory Accesses

## Two-Way Chaining Scheme

- It can be thought of as two chained hashing data structures
- We have two pseudo random hash functions $h_1$ and $h_2$
- Key x reside in either block $h_1(x)$ of hash table one or block $h_2(x)$ of hash table two
- New keys are inserted in the block with the smallest number of keys, with ties broken such that keys go to table one

# Lookup Using Two Parallel External Memory Accesses

## Two-Way Chaining Scheme

- It can be thought of as two chained hashing data structures
- We have two pseudo random hash functions $h_1$ and $h_2$
- Key x reside in either block $h_1(x)$ of hash table one or block $h_2(x)$ of hash table two
- New keys are inserted in the block with the smallest number of keys, with ties broken such that keys go to table one

## Analysis

- The probability of an insertion causing an overflow is $N/2^{2^{\Omega}(1-\alpha)B}$

# Lookup Using Two Parallel External Memory Accesses

## Two-Way Chaining Scheme

- It can be thought of as two chained hashing data structures
- We have two pseudo random hash functions $h_1$ and $h_2$
- Key x reside in either block $h_1(x)$ of hash table one or block $h_2(x)$ of hash table two
- New keys are inserted in the block with the smallest number of keys, with ties broken such that keys go to table one

## Analysis

- The probability of an insertion causing an overflow is $N/2^{2^{\Omega}(1-\alpha)B}$
- The effect of deletions does not appear to have been analyzed

# Resizing Hash Tables

Keep $\alpha$ in a certain interval to have a good external memory utilization

# Resizing Hash Tables

Keep $\alpha$ in a certain interval to have a good external memory utilization

**The challenge**

Rehash to the new table without an expensive reorganization of the old hash table

# Resizing Hash Tables

Keep $\alpha$ in a certain interval to have a good external memory utilization

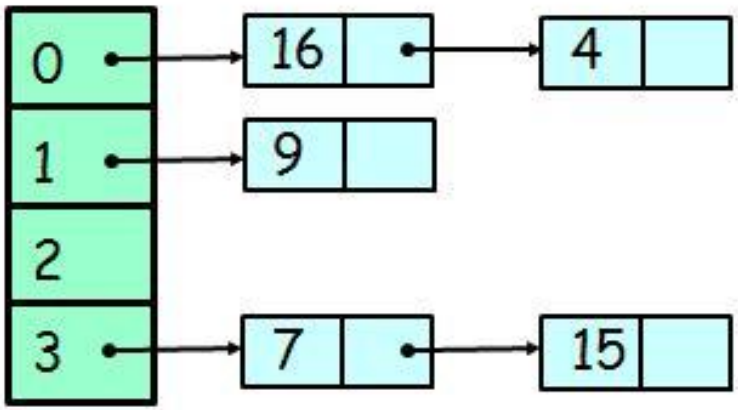
## The challenge

Rehash to the new table without an expensive reorganization of the old hash table

## The Solution

- Choosing a new convenient hash function
- This requires a especial random permutation of the keys
- For this task we require $\Theta(\frac{N}{B} log_{\frac{M}{B}}^{\frac{N}{B}})$ I/Os
- $N = (M/B)^{o(B)} \Longrightarrow$ O(N)I/Os
- $\Theta(N)$ updates between two rehashes

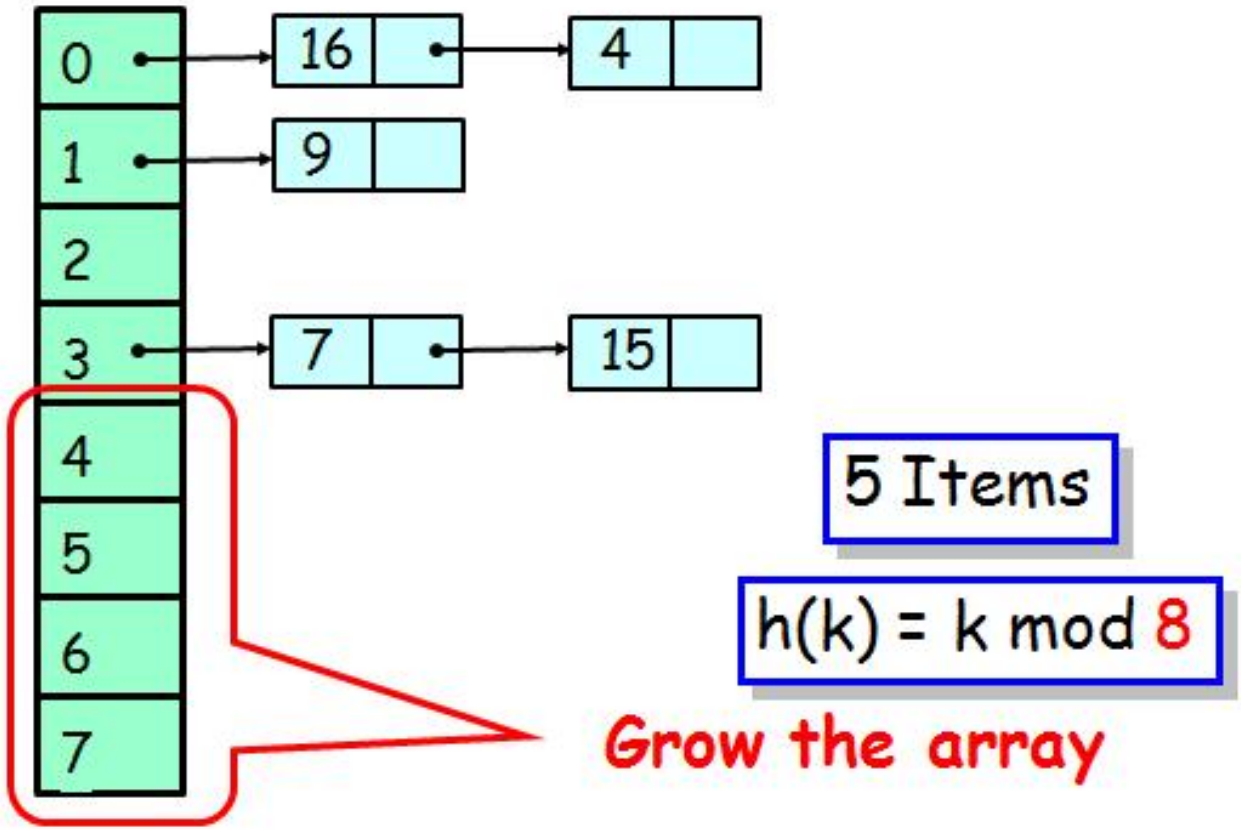
# Resizing Hash Tables Example



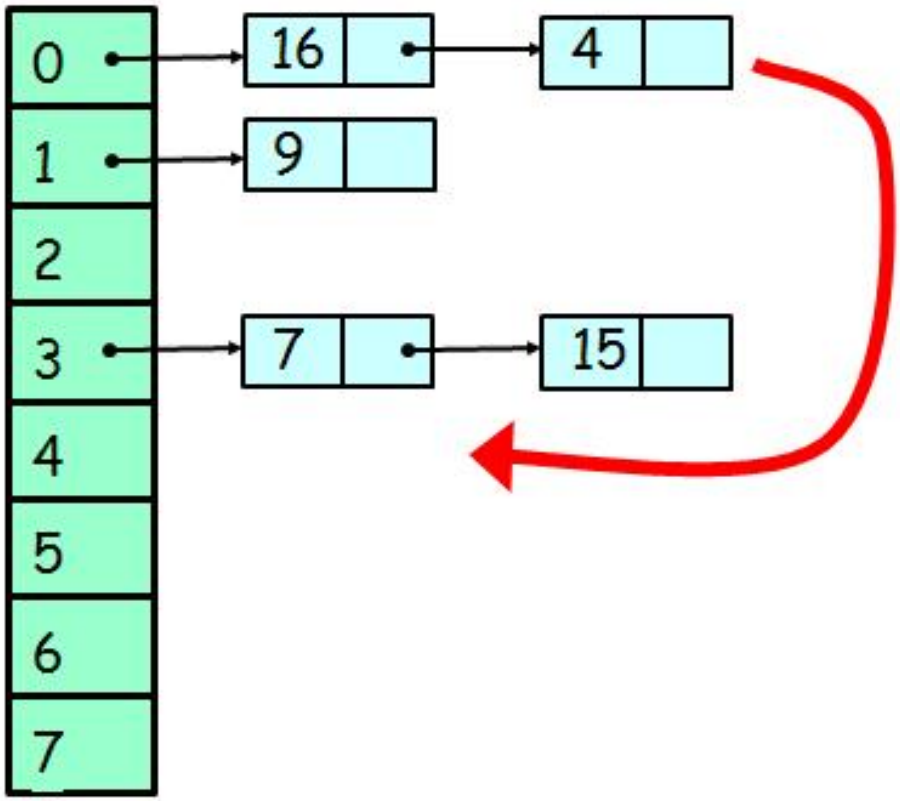

5 Items

$h(k) = k \bmod 4$

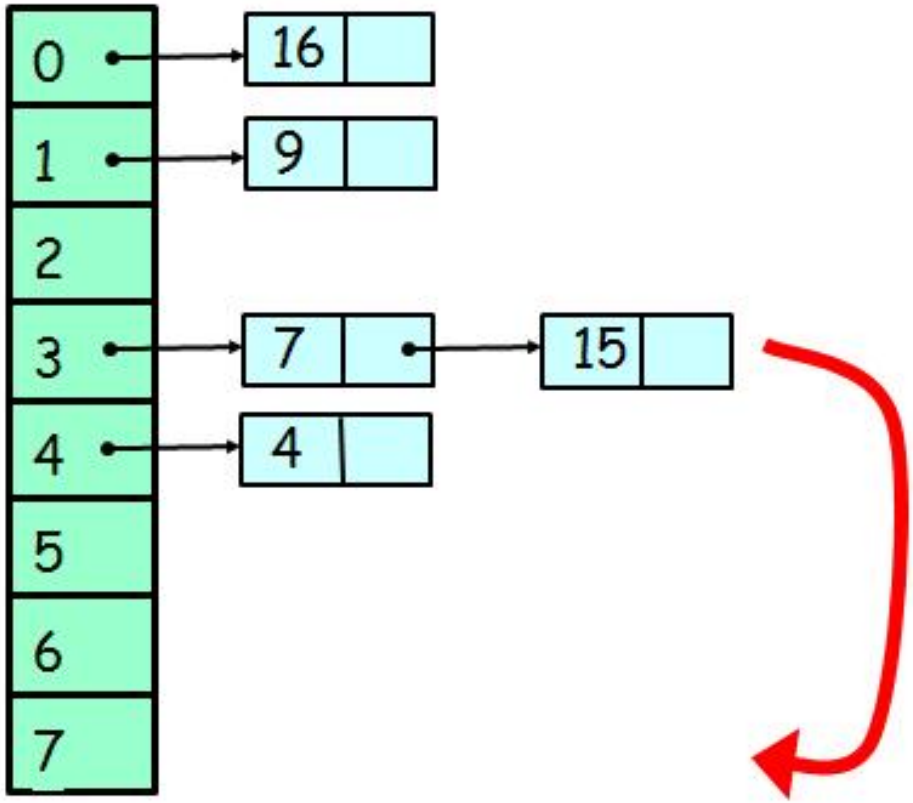
# Resizing Hash Tables Example



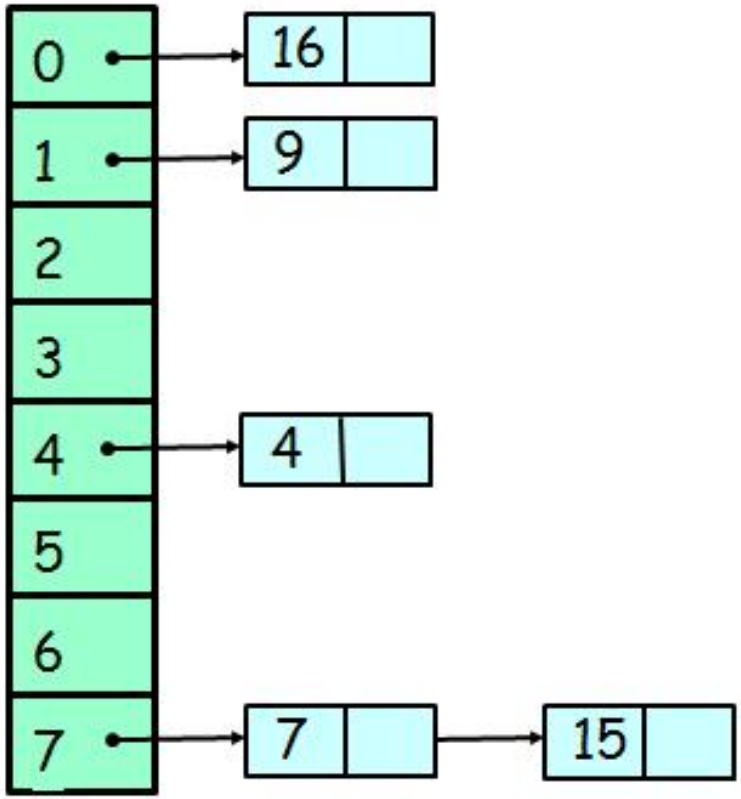
# Resizing Hash Tables Example



# Resizing Hash Tables Example



# Resizing Hash Tables Example



# Resizing Hash Tables (continue)

## Linear Hashing

# Resizing Hash Tables (continue)

## Linear Hashing

### The Basic Idea for Hashing to a Range of Size r

- Extract b $= \lceil log \rceil$ bits from a mother hash function
- If b bits encode an integer k less than r, this is used as the hash value
- Otherwise the hash function value $k - 2^{b-1}$ is returned
- Expand the size of the hash table by one block (increasing r by one)
- All keys that hash to the new block r+1 previously hashed to block $r + 1 - 2^{b-1}$
- Decreasing the size of the hash table is done in a symmetric manner

# Resizing Hash Tables (continue)

## Linear Hashing

### The Basic Idea for Hashing to a Range of Size r

- Extract b $= \lceil log \rceil$ bits from a mother hash function
- If b bits encode an integer k less than r, this is used as the hash value
- Otherwise the hash function value $k - 2^{b-1}$ is returned
- Expand the size of the hash table by one block (increasing r by one)
- All keys that hash to the new block r+1 previously hashed to block $r + 1 - 2^{b-1}$
- Decreasing the size of the hash table is done in a symmetric manner

### The Main Problem

When r is not a power of 2, the keys are not mapped uniformly to the range

- Dynamization Techniques

# The Logarithmic Method

# The Logarithmic Method

## The Problem Must Be **Decomposable**

- Split the set S of elements into disjoint subsets $S_1, ..., S_k$
- Create a (static) data structure for each of them
- Queries on the whole set can be answered by querying each of these data structures

# The Logarithmic Method

## The Problem Must Be **Decomposable**

- Split the set S of elements into disjoint subsets $S_1, ..., S_k$
- Create a (static) data structure for each of them
- Queries on the whole set can be answered by querying each of these data structures

## Examples of Decomposable Problems

Dictionaries and Priority Queues

# The Logarithmic Method (continue)

Obtain data structures with **insertion** and **query** operations

# The Logarithmic Method (continue)

Obtain data structures with **insertion** and **query** operations

## The Basic Idea

- Maintain a collection of data structures of different sizes
- Merge periodically a number data structures into one
- keep the number of data structures to be queried low
- In internal memory,the number of data structures is O(logN)

# The Logarithmic Method (continue)

Obtain data structures with **insertion** and **query** operations

## The Basic Idea

- Maintain a collection of data structures of different sizes
- Merge periodically a number data structures into one
- keep the number of data structures to be queried low
- In internal memory,the number of data structures is O(logN)

## The External Memory Version of the Logarithmic Method

- The number of data structures is decreased to $O(log_N^B)$
- Insertions are done by rebuilding the first static data structure
- The invariant is that the ith data structure should have size no more than $B^i$
- If this size is reached, it is merged with the i+1st data structure

# The Logarithmic Method (continue)

## Analysis

- Insert N elements, each element is part of a rebuilding $O(Blog_B^N)$ times
- Building a static data structure for N elements uses $O(\frac{N}{B}log_B^k N)$ I/Os
- The total amortized cost of inserting an element is $O(log_B^{k+1} N)$ I/Os
- Queries take $O(Blog_B^N)$ times more I/Os than queries in the corresponding static data structures

# Global Rebuilding

- Some data structures for sets support **deletions**, but do not recover the space occupied by deleted elements
- For example, **weak delete**
- Keep the number of deleted elements at some fraction of the total number of elements is global rebuilding

# Global Rebuilding

- Some data structures for sets support **deletions**, but do not recover the space occupied by deleted elements
- For example, **weak delete**
- Keep the number of deleted elements at some fraction of the total number of elements is global rebuilding

## The Main Idea

- In a data structure of N elements, whenever $\alpha$N elements have been deleted,for some constant $\alpha$ 0, the entire data structure is rebuilt
- The cost of rebuilding is at most a constant factor higher than the cost of inserting $\alpha$N elements
- The amortized cost of global rebuilding can be charged to the insertions of the deleted elements

خدایا

آرامشی عطا فرما تا بپذیرم آنچه که نمی‌توانم تغییر دهم و شهامتی تا تغییر دهم آنچه را که می‌توانم و دانشی که بدانم تفاوت آن دو را

دکتر علی شریعتی

**Thanks**

**Have a Good Day**