

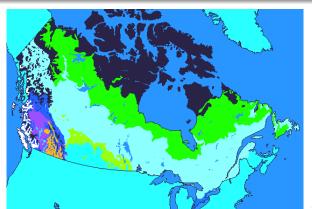
Computational Geometry

Doubly Connected Edge List (DCEL)

Computing the Overlay of Two Subdivisions

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- We have solved the easiest case of the map overlay problem, where the two maps are networks represented as collections of line segments.
- In general, maps have a more complicated structure: they are subdivisions of the plane into labeled regions.



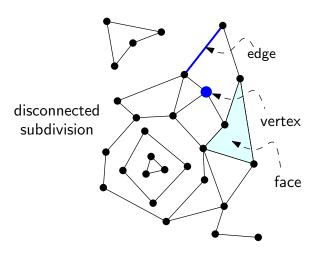


Doubly Connected Edge List (DCEL)

- Before we can give an algorithm for computing the overlay of two subdivisions, we must develop a suitable representation for a subdivision.
- Storing a subdivision as a collection of line segments is not such a good idea.
- Operations like reporting the boundary of a region would be rather complicated.
- Add topological information: which segments bound a given region, which regions are adjacent, and so on.



Doubly Connected Edge List (DCEL)





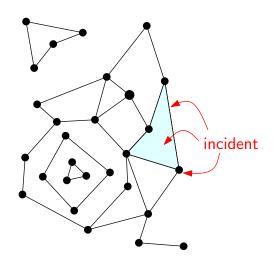
Doubly Connected Edge List (DCEL)

Computing the Overlay of Two Subdivisions

Complexity of a subdivision

#faces+#edges+#vertices.







Doubly Connected Edge List (DCEL)

Computing the Overlay of Two Subdivisions

Complexity of a subdivision #faces+#edges+#vertices.

What kind of queries?

- What is the face containing a given point? (TOO MUCH!)
- Walking around the boundary of a given face,
- Find the face from an adjacent one if we are given a common edge,
- Visit all the edges around a given vertex.

The representation that we shall discuss supports these operations. It is called the doubly-connected edge list (DCEL).



Computational Geometry

Doubly Connected Edge List (DCEL)

DCEL

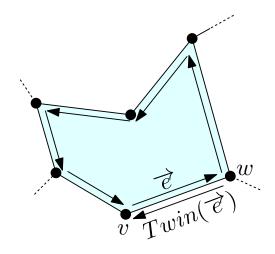
DCEL contains:

- a record for each edge,
- a record for each vertex,
- a record for each face,
- plus attribute information.



Computational Geometry

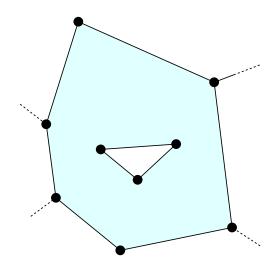
Doubly Connected Edge List (DCEL)





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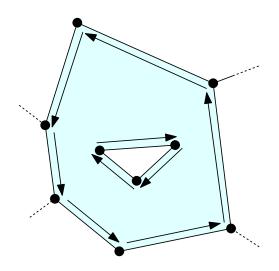
Computing the Overlay of Two Subdivisions





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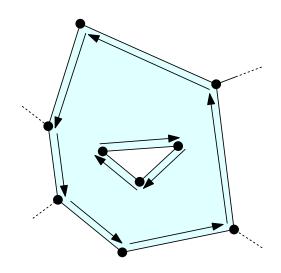
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Doubly Connected Edge List (DCEL)

Computing the Overlay of Two Subdivisions





Doubly Connected Edge List (DCEL)

Computing the Overlay of Two Subdivisions

DCEL

DCEL contains:

- a record for each vertex,
 - \bigcirc *Coordinates*(v): the coordinates of v,
 - ② IncidentEdge(v): a pointer to an arbitrary half-edge that has v as its origin.
- a record for each face,
 - OuterComponent(f): to some half-edge on its outer boundary (nil if unbounded),
 - InnerComponents(f): a pointer to some half-edge on the boundary of the hole, for each hole.
- a record for each half-edge \overrightarrow{e} ,
 - \bigcirc Origin(\overrightarrow{e}): a pointer to its origin,
 - 2 $Twin(\overrightarrow{e})$ a pointer to its twin half-edge,
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 - $Next(\overrightarrow{e})$ and $Prev(\overrightarrow{e})$: a pointer to the next and previous edge on the boundary of $IncidentFace(\overrightarrow{e})$.



Computational Geometry

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DCFL

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Computational Geometry

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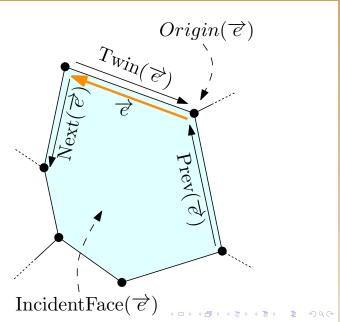
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Computational Geometry

Doubly Connected Edge List (DCEL)

DCEL



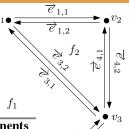


Computational Geometry

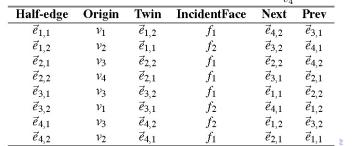
Doubly Connected Edge List (DCEL)

DCEL:Example

Vertex	Coordinates	IncidentEdge
ν_1	(0,4)	$ec{e}_{1,1}$
v_2	(2,4)	$\vec{e}_{4,2}$
v_3	(2,2)	$\vec{e}_{2,1}$
v_4	(1,1)	$ec{e}_{2,2}$



Face	OuterComponent	InnerComponents
f_1	nil	$ec{e}_{1,1}$
f_2	$ec{e}_{4,1}$	nil
	·	





Computational Geometry

Doubly Connected Edge List (DCEL)

DCEL

Time complexity of queries?

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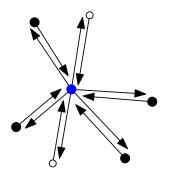
Computational Geometry

Doubly Connected Edge List (DCEL)

DCEL

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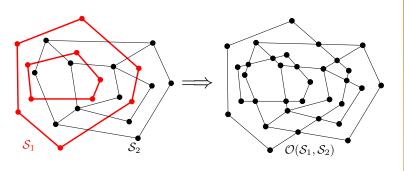
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Computational Geometry

Doubly Connected Edge List (DCEL)





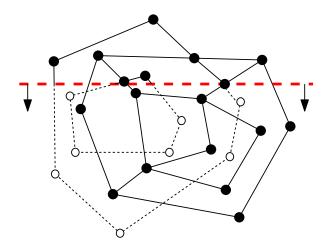
Computational Geometry

Doubly Connected Edge List (DCEL)

Computing the Overlay of Two Subdivisions

Main Idea in computing $\mathcal{O}(\mathcal{S}_1,\mathcal{S}_2)$

- Copy DCEL of S_1 and S_2 into new DCEL (Not a Valid DCEL).
- Make the new DCEL valid.



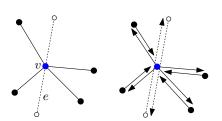


Computational Geometry

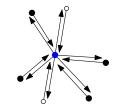
Doubly Connected Edge List (DCEL)

Updating half-edges

the geometric situation and the two doubly-connected edge lists before handling the intersection



the doubly-connected edge list after handling the intersection

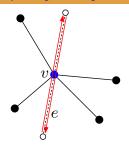


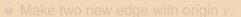


Computational Geometry

Doubly Connected Edge List (DCEL)

Updating half-edges





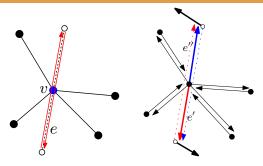
- Set Twin of new edges.
- Set Next() of the two new half-edges.
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Doubly Connected Edge List (DCEL)

Updating half-edges



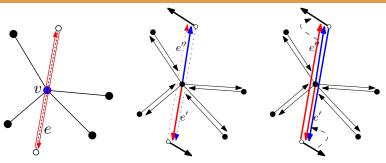
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Doubly Connected Edge List (DCEL)

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Edge List (DCEL)

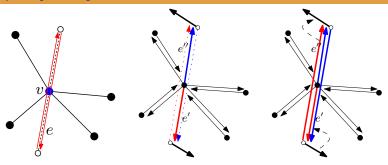
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Computational Geometry

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Updating half-edges



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- Computational Geometry
- Doubly Connected Edge List (DCEL)
- Computing the Overlay of Two Subdivisions

- Make two new edge with origin v.
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Updating half-edges

Fix the situation around v

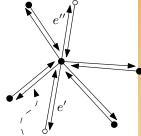
- The half-edge for e' that has v as its destination must be linked to the first half-edge, seen clockwise from e', with v as its origin.
- The half-edge for e' with v as its origin must be linked to the first counterclockwise half-edge with v as its destination
- The same for e''.
- Time complexity: $\mathcal{O}(m)$ (m: degree of v).



Computational Geometry

Doubly Connected Edge List (DCEL)

Computing the Overlay of Two Subdivisions



first clockwise half-edge from e^\prime with v as its origin

Updating half-edges

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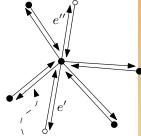
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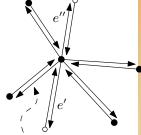
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Computing the Overlay of Two Subdivisions



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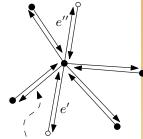
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Computational Geometry

Doubly Connected Edge List (DCEL)

Computing the Overlay of Two Subdivisions



first clockwise half-edge from e' with v as its origin

Computing the Overlay of Two Subdivisions Updating faces

Updating Faces:

- Create a face record for each $f \in \mathcal{O}(\mathcal{S}_1, \mathcal{S}_2)$.
- Set OuterComponent(f) and InnerComponent(f).



Computational Geometry

Doubly Connected Edge List (DCEL)

Updating faces

- # faces= # outer boundaries +1 (unbounded face).
- From half-edges we can construct the boundaries
- To determine weather the boundary is outer boundary or boundary of a hole:



Computational Geometry

Doubly Connected Edge List (DCEL)

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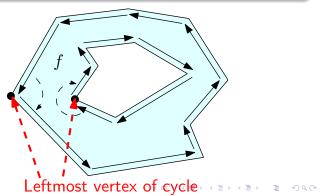


Computational Geometry

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Computational Geometry

Doubly Connected Edge List (DCEL)

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Which boundary cycles bound the same face?

- Construct a graph \mathcal{G} .
- Every boundary cycle is a node in G.
- One node for the imaginary outer boundary of the unbounded face.
- Add an arc between two cycles if and only if one of the cycles is the boundary of a hole and the other cycle has a half-edge immediately to the left of the leftmost vertex of that hole cycle.
- If there is no half-edge to the left of the leftmost vertex of a cycle, then the node representing the cycle is linked to the node of the unbounded face



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Computational Geometry

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Computing the Overlay of Two Subdivisions Updating faces

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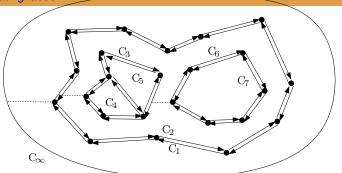
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Computational Geometry

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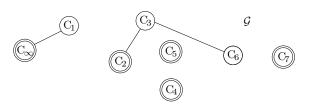
Updating faces





Computational Geometry

Doubly Connected Edge List (DCEL)



Computing the Overlay of Two Subdivisions Updating faces

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Lemma 2.5

Each connected component of the graph \mathcal{G} corresponds exactly to the set of cycles incident to one face.

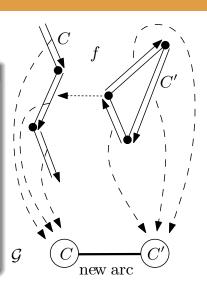
Computational Geometry

Doubly Connected Edge List (DCEL)

How can we Construct G?

Constructing G

- Similar to line segment intersection algorithm.
- Find the edge immediately to the left of point.
- We need a pointer from each edge to its boundary in G





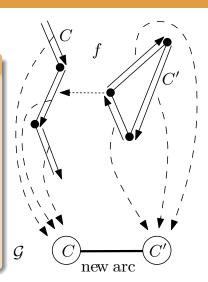
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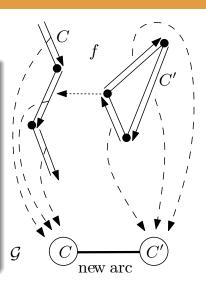
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Doubly Connected Edge List (DCEL)

Theorem 2.6

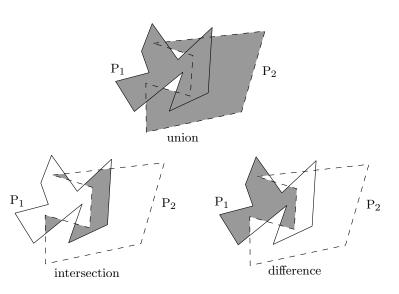
Let \mathcal{S}_1 be a planar subdivision of complexity n_1 , let \mathcal{S}_2 be a subdivision of complexity n_2 , and let $n:=n_1+n_2$. The overlay of \mathcal{S}_1 and \mathcal{S}_2 can be constructed in $\mathcal{O}(n\log n + k\log n)$ time, where k is the complexity of the overlay.



Computational Geometry

Doubly Connected Edge List (DCEL)

Application: Boolean Operations





Computational Geometry

Doubly Connected Edge List (DCEL)





Computational Geometry

Doubly Connected Edge List (DCEL)