On Algorithms for Computing the Diameter of a t-Spanner

Mohammad Farshi and Joachim Gudmundsson

Abstract

A t-spanner of a point set V is a undirected weighted graph G such that for each pair of points $p, q \in V$, there exist a path between p and q in G of length at most t times the distance between p and q (t-path). The diameter (t-diameter) of a t-spanner is the smallest integer d such that for any pair of vertices, there is a t-path in the graph between them containing at most d edges. As far as we know there is no known algorithm on how to compute the diameter of a t-spanner. In this paper we will give some algorithms for computing the diameter of a t-spanner. The time complexity of the most efficient algorithm is $\mathcal{O}(dmn)$, where n is the number of vertices, m is the number of edges and d is the diameter of the input graph, and it requires $\mathcal{O}(n)$ space.

AMS subject classifications: 05C85, 05C12, 68W01.

1 Introduction

Consider a set V of n points. A network on V can be modeled as an undirected graph G with vertex set V and an edge set E of size m where every edge $e = (u, v)$ has a weight $|uv|$. Let P be a path in G between u and v. The weight of P is defined as the sum of the weight of the edges of P. Let $t > 1$ be a real number. We say that G is a *t-spanner* for V, if for each pair of points $u, v \in V$, there exists a path in G of weight at most t times the distance between u and v. We call such a path a *t-path* between u and v.

Complete graphs represent ideal communication networks, but they are expensive to build; sparse spanners represent low-cost alternatives. Spanners find applications in robotics, network topology design, distributed systems, design of parallel machines, and many other areas and have been a subject of considerable research. Recently spanners found interesting practical applications in areas such as metric space searching [10] and broadcasting in communication networks [3]. For wireless ad hoc networks it is often desirable to have small diameter since it determines the maximum number of times a message has to be transmitted in a network. If a graph has diameter d then it is said to be a d-hop network. The problem of constructing spanners has received considerable attention from a theoretical perspective, see the surveys [4, 11].

2 Main Results

Let $G(V, E)$ be a t-spanner. The problem at hand is to compute the diameter of G. As far as we know there is no known algorithm on how to compute the diameter of a t-spanner, below we present a dynamic programming approach for the problem.

Notation. For each p and q in V, let $\delta_k(p, q)$ be the shortest path between p and q with at most k edges. In the case when we have more than one such a path we consider the path with minimum number of edges. Also assume that $L_k[p,q]$ is the length of $\delta_k(p,q)$. If there is no such path, we set $L_k[p,q]$ to ∞.

So the diameter of G is the smallest integer k such that $L_k[p,q] \leq t \cdot |pq|$, for all points p and q in V. Obviously

$$L_k(p,q) = \min\left\{ L_{k-1}(p,q), \min_{\binom{i+j=k}{r \in V \setminus \{p,q\}}} \{L_i(p,r) + L_j(r,q)\} \right\}.$$

See also [7, 8, 2, 5, 9, 1, 6].

References

[1] Abam, M. A., de Berg, M., Farshi, M., Gudmundsson, J., and Smid, M. Geometric spanners for weighted point sets. Manuscript, 2009.

[2] Agarwal, P. K., Klein, R., Knauer, C., and Sharir, M. Computing the detour of polygonal curves. Technical Report B 02-03, Fachbereich Mathematik und Informatik, Freie Universität Berlin, 2002.

[3] Alzoubi, K. M., Li, X.-Y., Wang, Y., Wan, P.-J., and Frieder, O. Geometric spanners for wireless ad hoc networks. *IEEE Transactions on Parallel and Distributed Systems 14*, 4 (2003), 408–421.

[4] Eppstein, D. Spanning trees and spanners. In *Handbook of Computational Geometry*, J.-R. Sack and J. Urrutia, Eds. Elsevier Science Publishers, Amsterdam, 2000, pp. 425–461.

[5] Har-Peled, S. A simple proof?, 2006. http://valis.cs.uiuc.edu/blog/?p=441.

[6] Khanban, A. A. *Basic Algorithms of Computational Geometry with Imprecise Input.* Ph.D. thesis, Imperial College London, University of London, UK, 2005.

[7] Mehlhorn, K., and Näher, S. *LEDA: A Platform for Combinatorial and Geometric Computing.* Cambridge University Press, Cambridge, UK, 2000.

[8] Narasimhan, G. Geometric spanner networks: Open problems. In *Invited talk at the* 1st Utrecht-Carleton Workshop on Computational Geometry (2002).

[9] Narasimhan, G., and Smid, M. *Geometric spanner networks.* Cambridge University Press, 2007.

[10] Navarro, G., and Paredes, R. Practical construction of metric t-spanners. In *ALENEX'03: Proceedings of the 5th Workshop on Algorithm Engineering and Experiments* (2003), SIAM Press, pp. 69–81.

[11] Smid, M. Closest point problems in computational geometry. In *Handbook of Computational Geometry*, J.-R. Sack and J. Urrutia, Eds. Elsevier Science Publishers, Amsterdam, 2000, pp. 877–935.