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Yazd University
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Winter 2015



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review

Imp.)
Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

Theoretical bounds

Applications

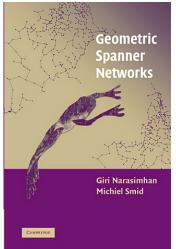
Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics

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Textbook:

Giri Narasimhan, Michiel Smid, **Geometric Spanner Networks**, CAMBRIDGE UNIVERSITY PRESS, 2007.





Geometric Spanner Networks

M. Farshi

Course Outline Textbook

ntroduction

Algorithms Review

Greedy Algorithm (Org. a Imp.)

Apx. Greedy Algorithm

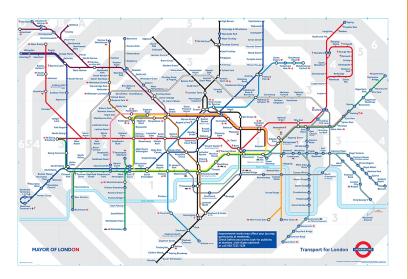
(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



London Underground Network





Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

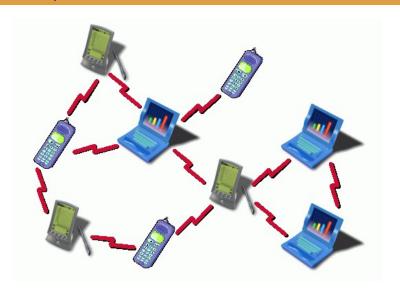
Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



Ad hoc Network



Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review

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Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

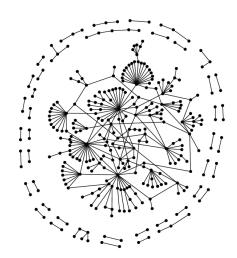
Theoretical

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

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Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

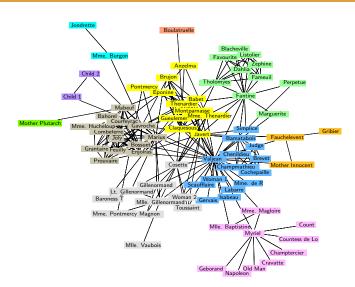
WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





A Social Network (Les Miserables characters)



Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review

Greedy Algorithm (Org. Imp.) Apx. Greedy Algorithm

(Ordered) ⊖-Grap Algorithm (Sink an Skip-list spanner)

WSPD-based Algo

Theoretica bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization







Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M Farshi

Course Outline

Introduction

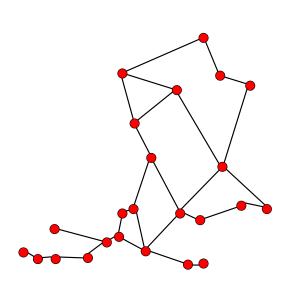
Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Imp.)

Algorithms Review

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review

Apx. Greedy Algorithm

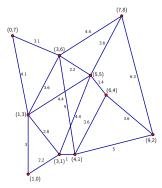
WSPD-based Algorithm

Theoretical

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics



Geometric Network

Weighted undirected graph G(V,E) s.t.

- $V \subset \mathbb{R}^d$.
- $\bullet \forall e = (u, v) \in E, wt(e) = |uv|.$



- Driving distance: 256 km. Actual distance: 198 km.
- $\frac{\text{Driving distance}}{\text{Actual distance}} = 1.27.$



Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algo

Theoretical

Applicat

Designing approximation algorithms with spanners Metric space searching Protein Visualization



- Driving distance: 180 km. Actual distance: 136 km.
- Driving distance =1.32.



Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algor (Ordered) ⊖-Grap Algorithm (Sink an Skip-list spanner)

WSPD-based Algo

Theoretical bounds

Applicat

Designing approximation algorithms with spanners Metric space searching Protein Visualization



• Driving distance: 143 km. Actual distance: 100 km.

Driving distance =1.43.



Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

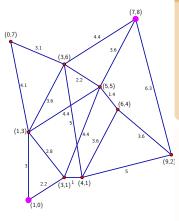
Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

Theoretical

oounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



Dilation (stretch factor)

between a pair of vertices=

Distance in the graph Euclidean distance

 of a network= maximum dilation between all pairs

t-spanner

A network with dilation at most t, or $\forall u, v \in V$, there is a path between u and v of length $\leq t \times |uv|$.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

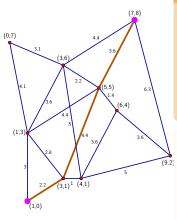
WSPD-based Algor

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





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Geometric Spanner Networks

M. Farshi

Course Outline Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

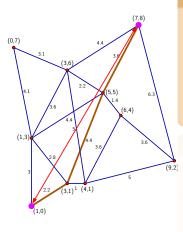
WSPD-based Algor

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

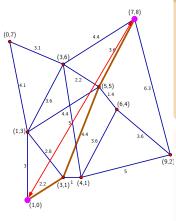
WSPD-based Algor

I heoretica bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

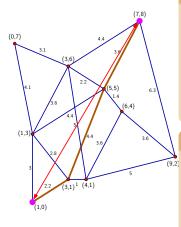
Sink Spanner WSPD-based Algor

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

WSPD-based Algo

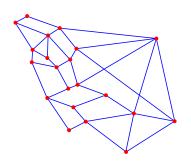
Theoretical bounds

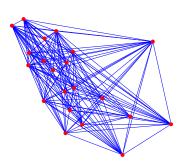
Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



 $(1+\varepsilon)$ -Spanners approximate the complete graphs with error ε .







Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review Greedy Algorithm (Org. and

Imp.)
Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

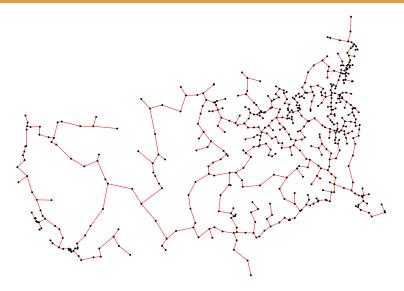
WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics



10-spanner for 532 US-cities





Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

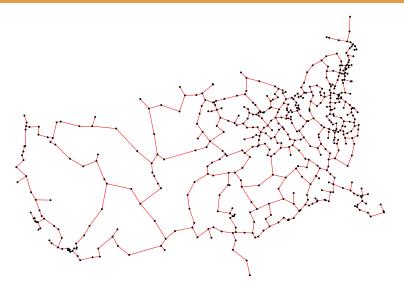
Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



5-spanner for 532 US-cities





Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review Imp.)

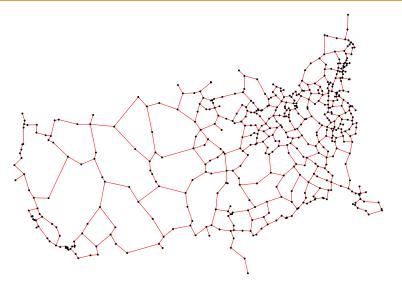
Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and

Sink Spanner WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



3-spanner for 532 US-cities





Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

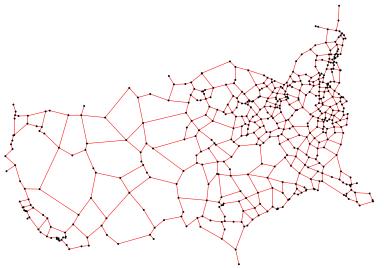
Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



2-spanner for 532 US-cities



Introduction Algorithms Review

Geometric Spanner Networks M Farshi

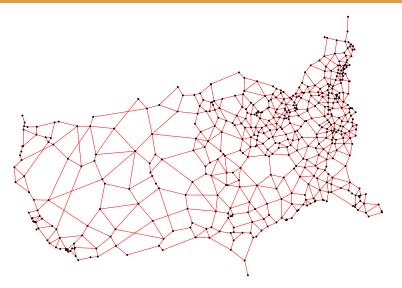
WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





1.5-spanner for 532 US-cities



Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review

Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

Theoretical bounds

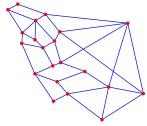
Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



How to compute a good spanner?

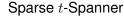




Given a set V and t > 1

Quality measurement:

- Number of edges (size)
- Weight (compared with MST)
- Maximum degree
- Diameter





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Geometric Spanner Networks

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Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner WSPD-based Algorithm

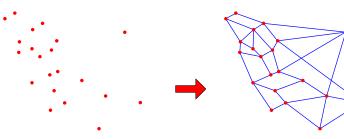
Theoretical bounds

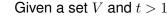
Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



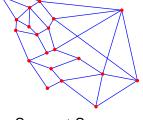
How to compute a good spanner?



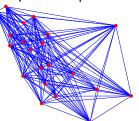


Quality measurement:

- Number of edges (size)
- Weight (compared with MST)
- Maximum degree
- Diameter



Sparse t-Spanner





Geometric Spanner Networks

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Course Outline

Introduction

Algorithms Review

Apx. Greedy Algorithm

Applications

Designing approximation algorithms with spanners Metric space searching



How to compute a good spanner?



Geometric Spanner Networks

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Course Outline

Introduction

Algorithms Review

Applications

Designing approximation algorithms with spanners Metric space searching

Research Topics

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Constructing sparse t-spanners:

- Greedy (Bern (1989) and Althöfer et al. (1993)).
- ⊖-graph (Clarkson (1987) and Keil (1988)).
- Ordered ⊖-graph (Bose et. al. (2004)).
- Well-Separated Pair Decomposition (Arya et. al. (1995)).



Geometric Spanner Networks

M. Farshi

ourse Outline

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and
Skin-list spanner)

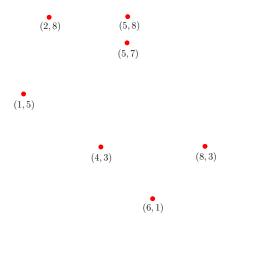
Sink Spanner WSPD-based Algorithm

Theoretical

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Algorithms Review Greedy Algorithm (Org. and Imp.)

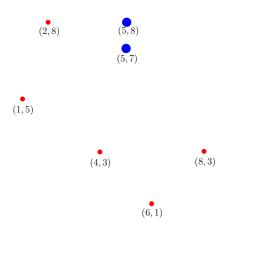
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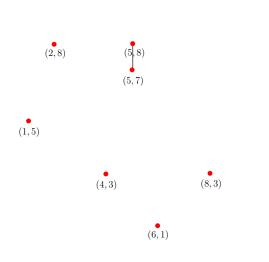
Sink Spanner WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization







Geometric Spanner Networks

M. Farshi

Course Outline

troduction

Algorithms Review
Greedy Algorithm (Org. and

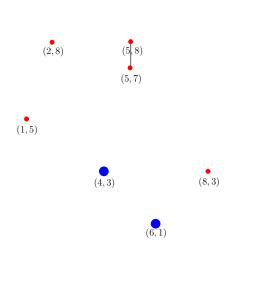
Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M Farshi

Algorithms Review Greedy Algorithm (Org. and

Imp.) Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and

Sink Spanner WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



Geometric Spanner Networks

M. Farshi

ourse Outline

ntroduction

Algorithms Review Greedy Algorithm (Org. and

Apx. Greedy Algorithm

(Ordered) ⊖-Graph

Algorithm (Sink and

Skip-list spanner)

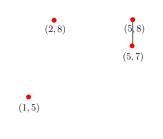
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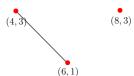
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Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization









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Algorithms Review Greedy Algorithm (Org. and Imp.)

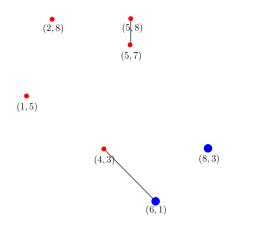
Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and

Sink Spanner WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

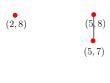
Sink Spanner

WSPD-based Algorithm

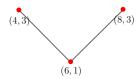
Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization











Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

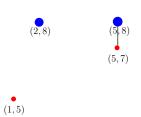
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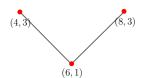
Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics





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Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and

Skip-list spanner)
Sink Spanner

WSPD-based Algorithm

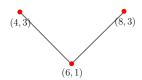
Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization











Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner
WSPD-based Algorithm

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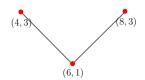
Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization











Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and

Skip-list spanner)
Sink Spanner

WSPD-based Algorithm

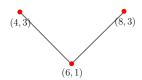
Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization











Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

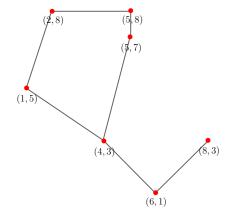
Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





ORG. GREEDY

```
Input: V and t > 1
Output: t-spanner G(V, E)
Sort pairs of points by non-decreasing order of distance;
E := \emptyset:
G := (V, E);
for each pair (u, v) of points (in sorted order) do
    if \mathsf{SHORTESTPATH}(G, u, v) > t \cdot |uv| then
         Add (u, v) to E;
    end
end
return G(V, E);
```

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Time Complexity: $\mathcal{O}(n^3 \log n)$. Storage Complexity: $\mathcal{O}(n^2)$.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algor

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

ORG. GREEDY

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```

Output: t-spanner G(V, E)

Sort pairs of points by non-decreasing order of distance;

4 D > 4 A > 4 B > 4 B >

```
E := \emptyset;

G := (V, E);
```

for each pair (u,v) of points (in sorted order) do

 $\label{eq:continuous_state} \begin{array}{l} \text{if } \mathsf{SHORTESTPATH}(G,u,v) > t \cdot |uv| \text{ then} \\ | \quad \mathsf{Add} \; (u,v) \text{ to } E; \end{array}$

end

end

return G(V, E);

Time Complexity: $\mathcal{O}(n^3 \log n)$.

Storage Complexity: $\mathcal{O}(n^2)$.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review Greedy Algorithm (Org. and Imp.)

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorit

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

ORG. GREEDY

```
\begin{array}{l} \textbf{Input: } V \text{ and } t > 1 \\ \textbf{Output: } t\text{-spanner } G(V, E) \\ \textbf{Sort pairs of points by non-decreasing order of distance;} \\ E := \emptyset; G := (V, E) ; \\ \textbf{for each pair } (u, v) \text{ of points (in sorted order) do} \\ & \quad \quad | \quad \quad
```

Number of shortest path queries: $\Theta(n^2)$.



Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithr (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Ale

wspb-based Alg

Theoretica bounds

Applications

Designing approximation algorithms with spanners Metric space searching

ORG. GREEDY

```
\begin{array}{l} \textbf{Input: } V \text{ and } t > 1 \\ \textbf{Output: } t\text{-spanner } G(V, E) \\ \textbf{Sort pairs of points by non-decreasing order of distance;} \\ E := \emptyset; G := (V, E) \ ; \\ \textbf{for } each \textit{pair } (u, v) \textit{ of points (in sorted order) } \textbf{do} \\ & \quad | \quad \textbf{if } \textbf{SHORTESTPATH}(G, u, v) > t \cdot |uv| \textbf{ then} \\ & \quad | \quad \textbf{Add } (u, v) \textbf{ to } E; \\ & \quad \textbf{end} \\ \textbf{end} \\ \textbf{return } G(V, E); \end{array}
```

Number of shortest path queries: $\Theta(n^2)$.

Observations:

- We only want to know if there is a t-path between u and v.
- The graph is only updated O(n) times.



Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algo

Theoretical bounds

Applications

Designing approximation algorithms with spanners

Metric space searching

IMP. GREEDY

```
Input: V and t > 1
Output: t-spanner G(V, E)
for each pair (u, v) \in V^2 do Set Weight(u, v) := \infty;
Sort pairs of points by non-decreasing order of distance;
E := \emptyset; G := (V, E);
for each pair (u, v) of points (in sorted order) do
     if Weight(u, v) \leq t \cdot |uv| then
          Skip (u, v);
     else
          Compute single source shortest path with source u;
          for each w do update Weight(u, w) and Weight(w, u);
          if Weight(u, v) \leq t \cdot |uv| then Skip (u, v);
          else Add (u, v) to E;
     end
end
return G(V, E);
```



Geometric Spanner Networks

M Farshi

Course Outline

ntroduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and
Skip-list spanner)

WSPD-based Algo

Theoretica bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algo

Theoretical

Applications

Designing approximation algorithms with spanners Metric space searching

Research Topics

Conjecture:

The running time of IMP. GREEDY is $\mathcal{O}(n^2 \log n)$.

Bose, Carmi, Farshi, Maheshvari and Smid (2008)

- The conjecture is wrong!
- They presented an algorithm which computes the greedy spanner in $\mathcal{O}(n^2 \log n)$ time (even for points from some metric spaces).

t-spanner Algorithm

Constant degree

t-spanner

Point set



Geometric Spanner Networks

M. Farshi

ourse Outline

ntroduction

Algorithms Review

Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm (Ordered) ⊖-Graph

Algorithm (Sink and Skip-list spanner) Sink Spanner WSPD-based Algorithm

Theoretical

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics

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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review

Imp.)
Apx. Greedy Algorithm

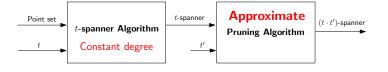
(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

WSPD-based Algorithm
Theoretical

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review

Apx. Greedy Algorithm

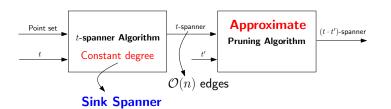
(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review

lmp.)

Apx. Greedy Algorithm

Algorithm (Sink and Skip-list spanner) Sink Spanner

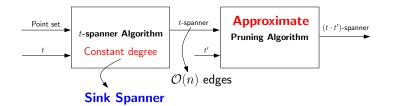
WSPD-based Algorithm

Theoretica bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics



Time Complexity: $\mathcal{O}(n\log^2 n)$

Storage Complexity: $\mathcal{O}(n)$.

$$t = 3, \Theta = \pi/6$$



Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review

Imp.)
Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and

Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



$$t = 3, \Theta = \pi/6$$



Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review

Imp.)
Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and

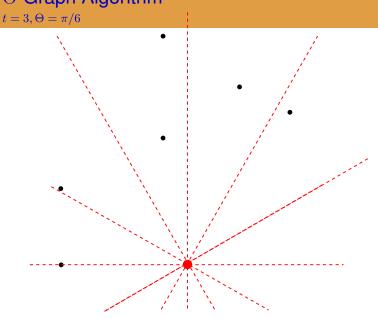
Skip-list spanner)
Sink Spanner
WSPD-based Algorithm

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M. Farshi

Algorithms Review

Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

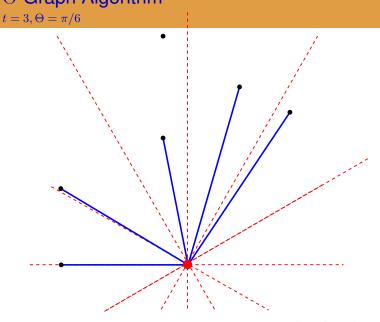
Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review

Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

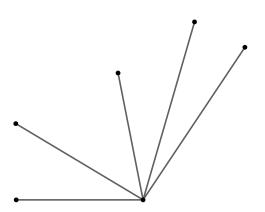
WSPD-based Algori

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

 $t = 3, \Theta = \pi/6$





Geometric Spanner Networks

M Farshi

Course Outline

ntroduction

Algorithms Review

Greedy Algorithm (Org. and

Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and

Skip-list spanner) Sink Spanner

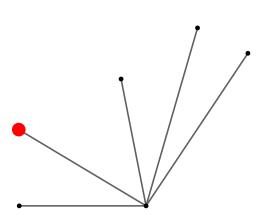
WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

$$t = 3, \Theta = \pi/6$$





Geometric Spanner Networks

M Farshi

Course Outline

ntroduction

Algorithms Review

Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



$$t = 3, \Theta = \pi/6$$



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review

Imp.)
Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

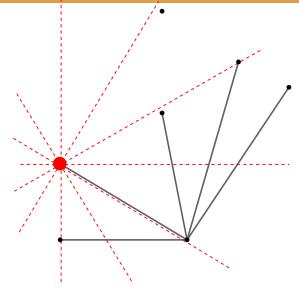
Sink Spanner WSPD-based Algorithm

WSPD-based Algo

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



 $t = 3, \Theta = \pi/6$



Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review

reedy Algorithm (Org. and

Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

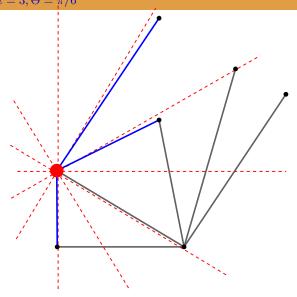
WSPD-based Algoriti

Theoretical bounds

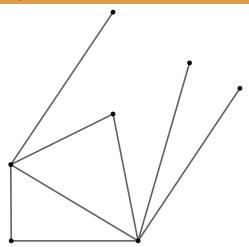
Applications

Designing approximation algorithms with spanners Metric space searching

Protein Visualization



$$t = 3, \Theta = \pi/6$$





Geometric Spanner Networks

M. Farshi

Algorithms Review

Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner

WSPD-based Algorithm

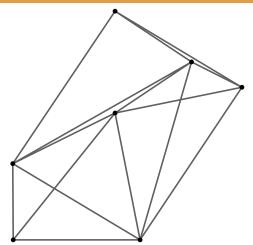
Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



 $t = 3, \Theta = \pi/6$





Geometric Spanner Networks

M. Farshi

Algorithms Review

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



Θ-GRAPH

```
Input: V and t > 1
Output: t-spanner G(V, E)
Set k:= the smallest integer such that t = \frac{1}{\cos \theta - \sin \theta} for
\theta = 2\pi/k;
E := \emptyset;
for each point u \in V do
     C_1, \ldots, C_k := non-overlapping cones with angle \theta
     and with apex at u;
     for each cone C<sub>i</sub> do
          Connect u to the closest point in C_i;
     end
end
return G(V, E);
```



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and

Skip-list spanner) Sink Spanner

WSPD-based Algor

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics

Time Complexity: $\mathcal{O}(n\log n)$. Storage Complexity: $\mathcal{O}(n)$.

Θ-GRAPH

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Input: V and t > 1
```

Output: t-spanner G(V, E)

Set k:= the smallest integer such that $t = \frac{1}{\cos \theta - \sin \theta}$ for $\theta = 2\pi/k$;

 $\theta = 2\pi/\kappa$ $E := \emptyset$;

for each point $u \in V$ do

 $C_1,\ldots,C_k:=$ non-overlapping cones with angle heta

and with apex at u;

for each cone C_i do

Connect u to the closest point in C_i ;

end

end

return G(V, E);

Time Complexity: $\mathcal{O}(n \log n)$. Storage Complexity: $\mathcal{O}(n)$.





Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Algorithm (Sink and Skip-list spanner)

WSPD-based Algorithm

Theoretical

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Variants of ⊖-Graph Algorithm

Ordered Θ -Graph- $\mathcal{O}(\log n)$ maximum degree

Same as the Θ -graph algorithm, except we add points one by one in a special order.

Random Ordered Θ -Graph– $\mathcal{O}(\log n)$ spanner diameter

We add points one by one in a random order.

Sink Spanner-bounded degree

Decrease the degree of nodes by replacing some edges by paths within other nodes.

Skip-List Spanner– $\mathcal{O}(\log n)$ spanner diameter

Decrease the diameter of Θ -graph by adding some extra edges.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review

Greedy Algorithm (Org. and

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and

Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and
Skip-list spanner)

WSPD-based Algorithm

Theoretical

Applicati

Designing approximation algorithms with spanners
Metric space searching
Protein Visualization

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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and
Skip-list spanner)

Sink Spanner WSPD-based Algorithm

WSPD-based Algorithm

bounds

Applicati

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Variants of Θ-Graph Algorithm

Ordered Θ -Graph– $\mathcal{O}(\log n)$ maximum degree

Same as the $\Theta\text{-graph}$ algorithm, except we add points one by one in a special order.

Random Ordered Θ -Graph- $\mathcal{O}(\log n)$ spanner diameter

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Sink Spanner-bounded degree

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Skip-List Spanner– $\mathcal{O}(\log n)$ spanner diameter

Decrease the diameter of Θ -graph by adding some extra edges.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm
(Ordered) ⊖-Graph
Algorithm (Sink and
Skip-list spanner)

WSPD-based Algorithm

WSPD-based Algorithm

bounds

Applicat

Designing approximation algorithms with spanners Metric space searching

Research Topics

05/00

Sink Spanner

A variant of ⊖-graph with bounded degree

Input: V and t > 1

Output: t-spanner G(V, E)

Construct a directed \sqrt{t} -spanner \overrightarrow{G} with bounded

out-degree;

for each point $q \in V$ do

Replace the "star" pointing to q by a \sqrt{t} -q-sink spanner

end

return G(V, E);

Time Complexity: $\mathcal{O}(n\log n)$ Storage Complexity: $\mathcal{O}(n)$



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

px. Greedy Algorithm Ordered) ⊖-Graph Ilgorithm (Sink and

Sink Spanner

WSPD-based Algorit

Theoretica bounds

Applications

Designing approximation algorithms with spanners

Metric space searching

Protein Visualization

Sink Spanner

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Input: V and t > 1

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Construct a directed \sqrt{t} -spanner \overrightarrow{G} with bounded out degree:

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end

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Time Complexity: $O(n \log n)$ Storage Complexity: O(n).



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorith
Ordered) ⊖-Graph
Algorithm (Sink and

Sink Spanner

WSPD-based Algorit

Theoretical counds

Applicati

Designing approximation algorithms with spanners
Metric space searching

Skip-List Spanner

A variant of Θ -graph with $\mathcal{O}(\log n)$ spanner diameter

```
Input: V and t > 1
Output: t-spanner G(V, E)
Set V_0 := V; i := 1;
while V_{i-1} \neq \emptyset do
    V_i contains each points of V_{i-1} with probability 1/2;
end
for each i do
    Construct a t-spanner G_i(V_i, E_i) using the \Theta-graph
     algorithm:
end
E = \bigcup_i E_i;
return G(V, E);
```



Geometric Spanner Networks

M Farshi

Course Outline

Algorithms Review

Sink Spanner

Applications

Designing approximation algorithms with spanners Metric space searching

Skip-List Spanner

A variant of Θ -graph with $\mathcal{O}(\log n)$ spanner diameter

```
Input: V and t > 1
```

Output: t-spanner G(V, E)

Set $V_0 := V$; i := 1; while $V_{i-1} \neq \emptyset$ do

 V_i contains each points of V_{i-1} with probability 1/2;

end

for each i do

Construct a t-spanner $G_i(V_i, E_i)$ using the Θ -graph algorithm;

end

 $E = \cup_i E_i;$ return G(V, E);

Time Complexity: $O(n \log n)$ Storage Complexity: O(n).



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithn Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner

WSPD-based Algorith

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Well Separated Pair Decomposition (WSPD)

Well Separated Pair

 $A,B\subset\mathbb{R}^d$ are s-well separated (s>0), if \exists disjoint balls, D_A and D_B such that



Geometric Spanner Networks

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Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

WSPD-based Al

Theoretical

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



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Geometric Spanner Networks

M Farshi

Course Outline

Algorithms Review

Apx. Greedy Algorithm

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

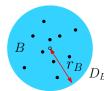


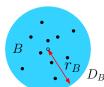


Well Separated Pair:

 $A, B \subset \mathbb{R}^d$ are s-well separated (s > 0), if \exists disjoint balls, D_A and D_B such that

- $A \subseteq D_A$ and $B \subseteq D_B$.







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Geometric Spanner Networks

M Farshi

Course Outline

Algorithms Review

Apx. Greedy Algorithm

WSPD-based Algorithm

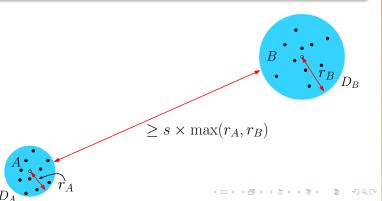
Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Well Separated Pair:

 $A,B\subset\mathbb{R}^d$ are s-well separated (s>0), if \exists disjoint balls, D_A and D_B such that

- $A \subseteq D_A$ and $B \subseteq D_B$.
- $\mathbf{d}(D_A, D_B) \ge s \times \max(\mathrm{radius}(D_A), \mathrm{radius}(D_B)).$





Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



Let $V \subset \mathbb{R}^d$ and s > 0. A WSPD for V with respect to s is a set $\{(A_i, B_i)\}_{i=1}^m$ of pairs of non-empty subsets of V such that

- $\forall i, A_i$ and B_i are s-well separated,
- $\forall p, q \in V$, there is exactly one index i s. t.
 - $p \in A_i$ and $q \in B_i$ or
 - $q \in A_i$ and $p \in B_i$.

m: Size of WSPD.

Callahan & Kosaraju (1995)

For each set of n points, we can construct a WSPD of size $\mathcal{O}(s^d \cdot n)$ in $\mathcal{O}(n \log n)$ time using $\mathcal{O}(s^d \cdot n)$ space.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review

Greedy Algorithm (Org. and

Sink Spanner WSPD-based Algorithm

WSPD-based Algorit

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



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Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithn Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

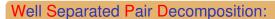
Sink Spanner WSPD-based Algor

WSPD-based Algorithm

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



Let $V \subset \mathbb{R}^d$ and s > 0. A WSPD for V with respect to s is a set $\{(A_i, B_i)\}_{i=1}^m$ of pairs of non-empty subsets of V such that

- $\forall i, A_i$ and B_i are s-well separated,
- $\forall p, q \in V$, there is exactly one index i s. t.
 - $p \in A_i$ and $q \in B_i$ or
 - $q \in A_i$ and $p \in B_i$.

m: Size of WSPD.

Callahan & Kosaraju (1995)

For each set of n points, we can construct a WSPD of size $\mathcal{O}(s^d \cdot n)$ in $\mathcal{O}(n \log n)$ time using $\mathcal{O}(s^d \cdot n)$ space.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithn Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

WSPD-based Algor

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Well Separated Pair Decomposition:

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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

WSPD-based Algorith

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

T.

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



WSPD-based Algorithm

WSPD Algorithm

Input: V and t > 1

Output: t-spanner G(V, E)

Set W := WSPD of V w.r.t. $s := \frac{4(t+1)}{t-1}$;

Set $E = \emptyset$;

for each $(A_i, B_i) \in \mathcal{W}$ do

Select an arbitrary node $u \in A_i$ and an arbitrary node

 $v \in B_i$;

Add edge (u, v) to E.

end

return G(V, E).

Time Complexity: $\mathcal{O}(n \log n)$. Storage Complexity: $\mathcal{O}(n)$.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skin-list spanner)

Sink Spanner WSPD-based Algorithm

WSPD-based Algorith

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skin-list spanner)

Sink Spanner WSPD-based Algorithm

WSPD-based Algoriti

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



Theoretical bounds

-	Size	Weight	Degree	Time
Greedy spanner	$\mathcal{O}(n)$	$\mathcal{O}(wt(\mathrm{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n^2 \log n)$
Apx. greedy spanner	$\mathcal{O}(n)$	$\mathcal{O}(wt(\mathrm{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n \log n)$
⊖-graph	$\mathcal{O}(n)$	$\Theta(n \cdot wt(\text{MST}))$	$\Theta(n)$	$\mathcal{O}(n \log n)$
O. ⊝-graph	$\mathcal{O}(n)$	$\mathcal{O}(n \cdot wt(\mathrm{MST}))$	$\mathcal{O}(\log n)$	$\mathcal{O}(n \log n)$
WSPD spanner	$\mathcal{O}(n)$	$\mathcal{O}(\log n \cdot wt(\text{MST}))$	$\Theta(n)$	$\mathcal{O}(n \log n)$
Sink-spanner	$\mathcal{O}(n)$	$\mathcal{O}(n \cdot wt(\mathrm{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n \log n)$
Skip-list spanner	$\mathcal{O}(n)^*$	$\Theta(n \cdot wt(\text{MST}))^*$	$\Theta(n)$	$\mathcal{O}(n\log n)^*$

(*): Expected with high probability



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)
Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization





Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review

Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Designing approximation algorithms with spanners

Traveling Salesperson Problem (TSP)

Find the shortest tour that visits each point exactly once and return to the starting point.





Geometric Spanner Networks

M. Farshi

Course Outline

troduction

Algorithms Review

reedy Algorithm (Org. ; np.) ox. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Alg

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching

rotein Visualizatio

Designing approximation algorithms with spanners

Traveling Salesperson Problem (TSP)

Find the shortest tour that visits each point exactly once and return to the starting point.

Known results:

- The problem is NP-hard even in \mathbb{R}^d .
- A 2-approximation algorithm for metric spaces by Rosenkrantz et al. (1977).
- A 1.5-approximation algorithm by Christofides et al. (1976).
- A PTAS ($(1 + \varepsilon)$ -approx. Alg.) for geometric case by Arora (1998) and Mitchell (1999).
- A PTAS for geometric case using spanners by Rao and Smith (1998).



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithr

Theoretica bounds

Applications

Designing approximation algorithms with spanners Metric space searching

rotein Visualiz

Designing approximation algorithms with spanners

Definition:

If G is a graph with vertex set P, then a tour of P in G is a (possibly non-simple) cycle in G that visits each point of P at least once.

Observation

For any t-spanner G for P, there is a tour of P in G, whose weight is at most $t \cdot wt(TSP(P))$.

Theorem (Rao and Smith, 1998)

Given a $(1+\varepsilon)$ -spanner of a set of n points with $\mathcal{O}(n)$ size and $\mathcal{O}(wt(\mathrm{MST}))$ weight, we can compute a $(1+\varepsilon)$ -approximation of $\mathrm{TSP}(P)$ in $\mathcal{O}(n\log n)$ time.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorit

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics

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Designing approximation algorithms with spanners

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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching

Research Topics

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Designing approximation algorithms with spanners

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S. B. Rao and W. D. Smith, **Approximating Geometrical Graphs via** "**Spanners" and "Banyans"**, STOC'98, pp. 540–550, 1998.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

(Ordered) Θ -Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

heoretical

bounds

Applications

Designing approximation algorithms with spanners Metric space searching

Research Topics

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Metric space searching

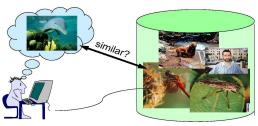


image database

Approximate proximity searching

- Multimedia information retrieval,
- Data mining,
- Pattern recognition,
- Machine learning,
- Computer vision and
- Biomedical databases.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and

Sink Spanner WSPD-based Algorithm

Theoretical

Applications

Designing approximation algorithms with spanners

Metric space searching

Protein Visualization

Research Topics

Metric space searching

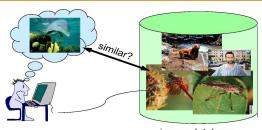


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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

px. Greedy Algorithm Ordered) ⊖-Graph Igorithm (Sink and kin-list spanner)

Sink Spanner WSPD-based Algorith

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching

Protein Visualization

Metric space searching

What is the role of spanners?

- A meter show the similarity between any two objects.
- But evaluating the distances are expensive.
- One way to speedup is computing the distance between any two objects and save them, but it need $\mathcal{O}(n^2)$ space (AESA).
- A t-spanner can be used as a sparse data structure to reduce the space.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

(Ordered) Θ -Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algor

Theoretical bounds

Applications

Designing approximation algorithms with spanners

Metric space searching

Protein Visualization



Metric space searching

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Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

WSPD-based Algor

Theoretical

Applications

Designing approximation algorithms with spanners

Metric space searching

Protein Visualization



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Geometric Spanner Networks

M. Farshi

ourse Outline

Introduction

Algorithms Review

Greedy Algorithm (Org. and Imp.)

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algor

heoretical ounds

Applications

Designing approximation algorithms with spanners

Metric space searching

Protein Visualiza



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G. Navarro, R. Paredes, and E. Chávez, **t-Spanners for metric space searching**, Data & Knowledge Engineering, pp. 820-854, 2007.



Geometric Spanner Networks

M. Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

VSPD-based Algori

heoretical ounds

Application

Designing approximation algorithms with spanners Metric space searching

Protein Visual



Protein Visualization





D. Russel and L. Guibas, **Exploring Protein Folding Trajectories Using Geometric Spanners**, Pacific Symposium on Biocomputing, pp. 40-51, 2005.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review

Greedy Algorithm (Org. an Imp.) Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching

Protein Visualization

Current and Future Works:

- Dynamic spanners (insert and remove nodes).
- Kinetic spanners (when points move and we want to maintain an spanner all the time).
- Fault-tolerant spanners (vertex/edge fault tolerant or region fault tolerant).
- Spanners among obstacles.
- Optimization problems.
- External memory (I/O efficient) algorithms for generating spanners.
- Experimental works on spanner algorithms.



Geometric Spanner Networks

M. Farshi

Course Outline
Textbook

Introduction

Algorithms Review
Greedy Algorithm (Org. and Imp.)

(Ordered) Θ -Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



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- Dynamic spanners (insert and remove nodes).
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- Spanners among obstacles.
- Optimization problems
- External memory (I/O efficient) algorithms for generating spanners.
- Experimental works on spanner algorithms.



Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algorithm

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization



بنياديز,

Geometric Spanner Networks

M Farshi

Course Outline Textbook

Introduction

Algorithms Review Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and

Algorithm (Sink and Skip-list spanner) Sink Spanner

WSPD-based Algor

bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization

Research Topics

- Dynamic spanners (insert and remove nodes).
- Kinetic spanners (when points move and we want to maintain an spanner all the time).
- Fault-tolerant spanners (vertex/edge fault tolerant or region fault tolerant).
- Spanners among obstacles.
- Optimization problems.
- External memory (I/O efficient) algorithms for generating spanners.
- Experimental works on spanner algorithms.

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Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algori

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching

Research Topics

- Dynamic spanners (insert and remove nodes).
- Kinetic spanners (when points move and we want to maintain an spanner all the time).
- Fault-tolerant spanners (vertex/edge fault tolerant or region fault tolerant).
- Spanners among obstacles.
- Optimization problems
- External memory (I/O efficient) algorithms for generating spanners.
- Experimental works on spanner algorithms.

بنياريز,

Geometric Spanner Networks

M Farshi

Course Outline Textbook

Introduction

Algorithms Review Greedy Algorithm (Org. and

Apx. Greedy Algorith (Ordered) ⊖-Graph Algorithm (Sink and

Skip-list spanner) Sink Spanner

WSPD-based Algor

Theoretica bounds

Applications

Designing approximation algorithms with spanners
Metric space searching

Research Topics

- Dynamic spanners (insert and remove nodes).
- Kinetic spanners (when points move and we want to maintain an spanner all the time).
- Fault-tolerant spanners (vertex/edge fault tolerant or region fault tolerant).
- Spanners among obstacles.
- Optimization problems.
- External memory (I/O efficient) algorithms for generating spanners.
- Experimental works on spanner algorithms.

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Geometric Spanner Networks

M Farshi

Course Outline

Introduction

Algorithms Review
Greedy Algorithm (Org. and

Apx. Greedy Algorithm (Ordered) ⊖-Graph Algorithm (Sink and

Sink Spanner
WSPD-based Algorith

VSPD-based Alg

Theoretica bounds

Applications

Designing approximation algorithms with spanners Metric space searching

Research Topics

- Dynamic spanners (insert and remove nodes).
- Kinetic spanners (when points move and we want to maintain an spanner all the time).
- Fault-tolerant spanners (vertex/edge fault tolerant or region fault tolerant).
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- Optimization problems.
- External memory (I/O efficient) algorithms for generating spanners.
- Experimental works on spanner algorithms.



Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review
Greedy Algorithm (Org. and

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner) Sink Spanner

neoretical

Annlication

Designing approximation algorithms with spanners Metric space searching







Geometric Spanner Networks

M. Farshi

Course Outline

ntroduction

Algorithms Review

Imp.)
Apx. Greedy Algorithm

(Ordered) ⊖-Graph Algorithm (Sink and Skip-list spanner)

Sink Spanner WSPD-based Algorithm

Theoretical bounds

Applications

Designing approximation algorithms with spanners Metric space searching Protein Visualization