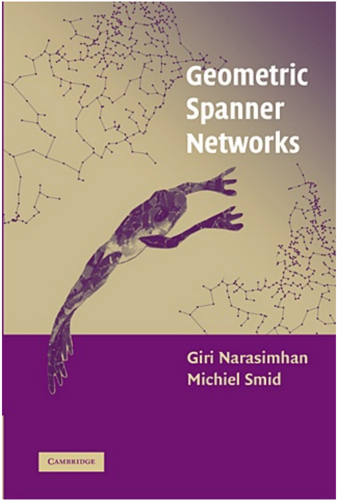
Geometric Spanner Networks

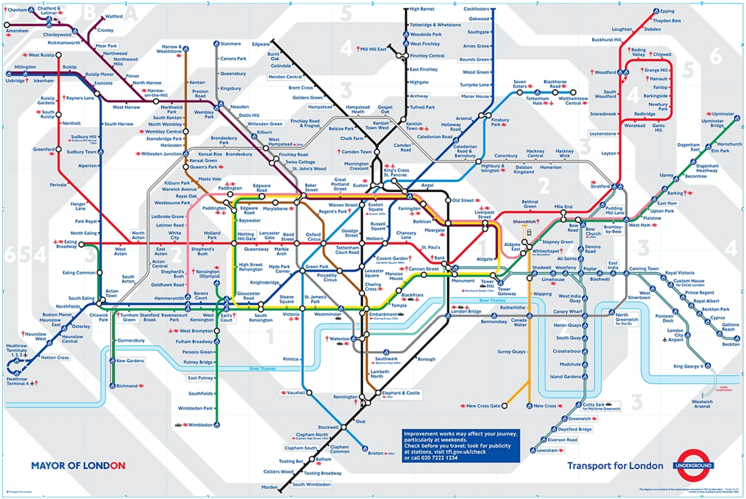
Spring 2014

Textbook:

Giri Narasimhan, Michiel Smid, **Geometric Spanner Networks**, CAMBRIDGE UNIVERSITY PRESS, 2007.

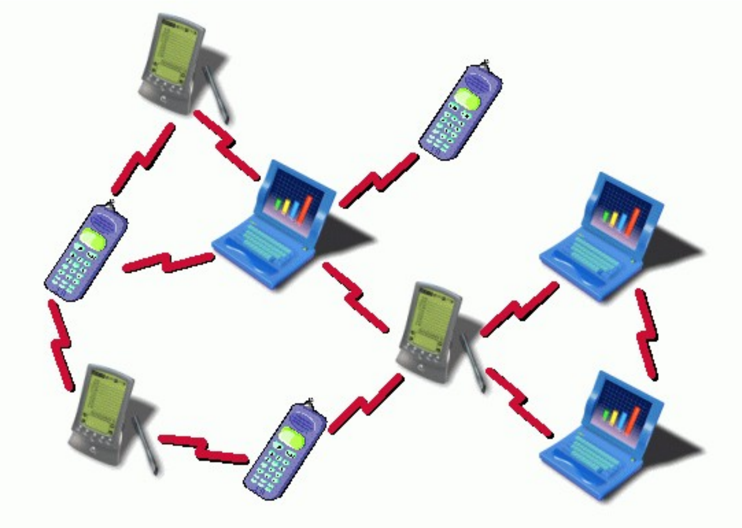


Example of networks



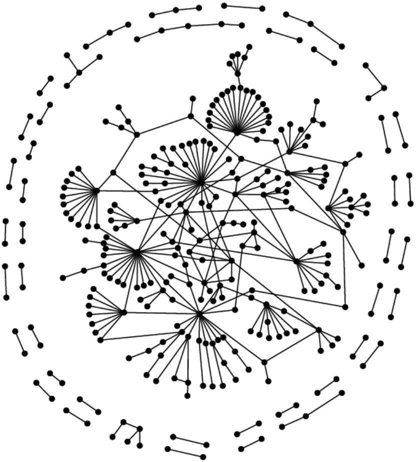
London Underground Network

Example of networks



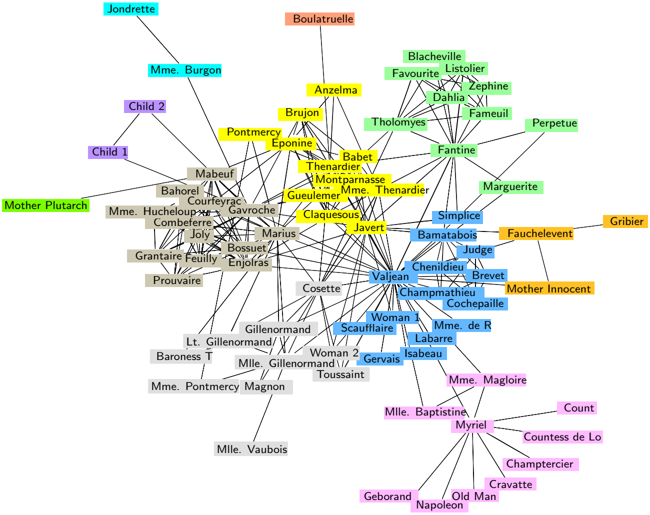
Ad hoc Network

Example of networks



Yeast Protein Interaction Network

Example of networks



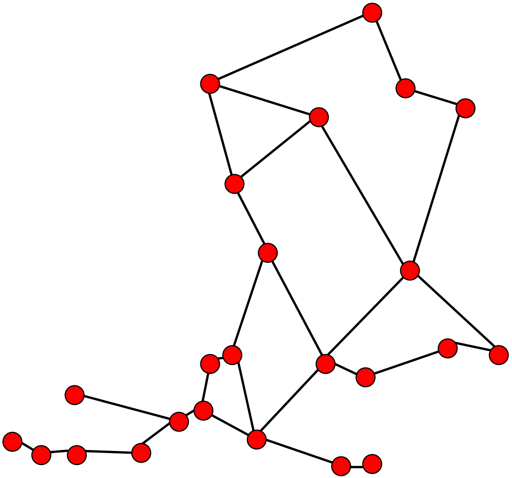
A Social Network (Les Miserables characters)

Geometric Network

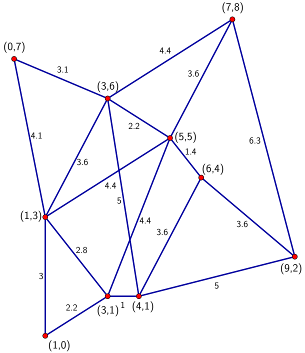
Geometric Network

Geometric Network

Geometric Network



Geometric Network



Geometric Network

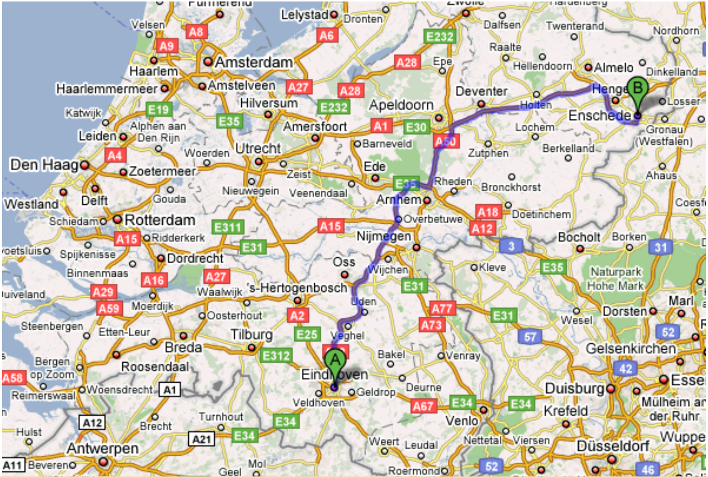
Weighted undirected graph $G(V, E)$ s.t.

- $V \subset \mathbb{R}^d$.
- $\forall e = (u, v) \in E, wt(e) = |uv|$.

Network Quality

- Driving distance: 256 km. Actual distance: 198 km.
- $\dfrac{\text{Driving distance}}{\text{Actual distance}} = 1.27.$

Network Quality

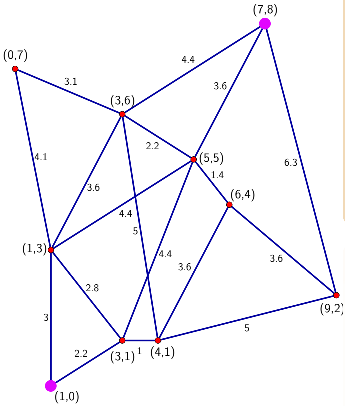


- Driving distance: 180 km.   Actual distance: 136 km.
- $\dfrac{\text{Driving distance}}{\text{Actual distance}} = 1.32.$

Network Quality

- Driving distance: 143 km. Actual distance: 100 km.
- $\dfrac{\text{Driving distance}}{\text{Actual distance}} = 1.43.$

Network Quality



Dilation (stretch factor)

- between a pair of vertices=

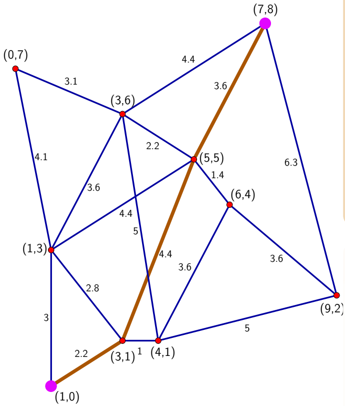
$$\frac{\text{Distance in the graph}}{\text{Euclidean distance}}$$

- of a network= maximum dilation between all pairs.

t-spanner

A network with dilation at most t, or $\forall u, v \in V$, there is a path between u and v of length $\leq t \times |uv|$. (t-path)

Network Quality



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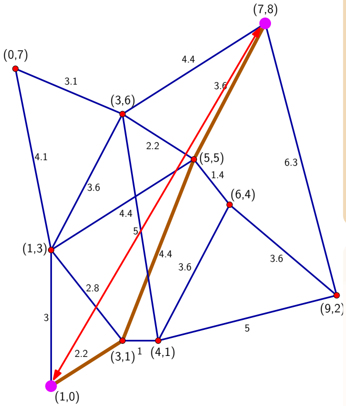

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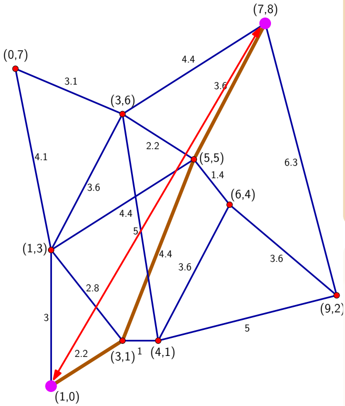

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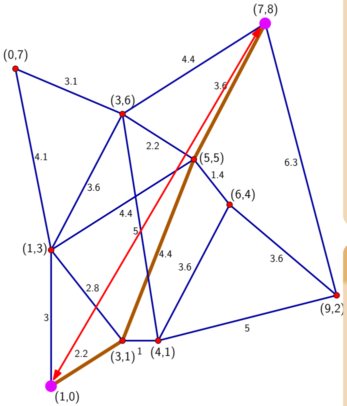

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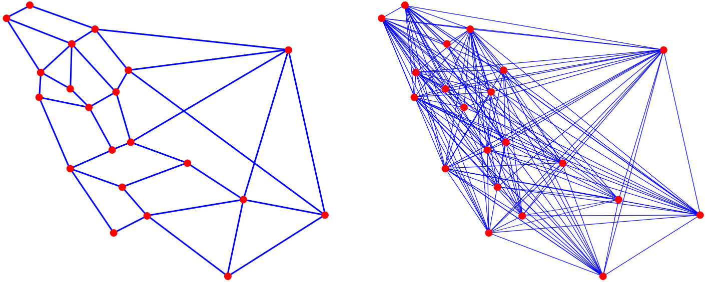
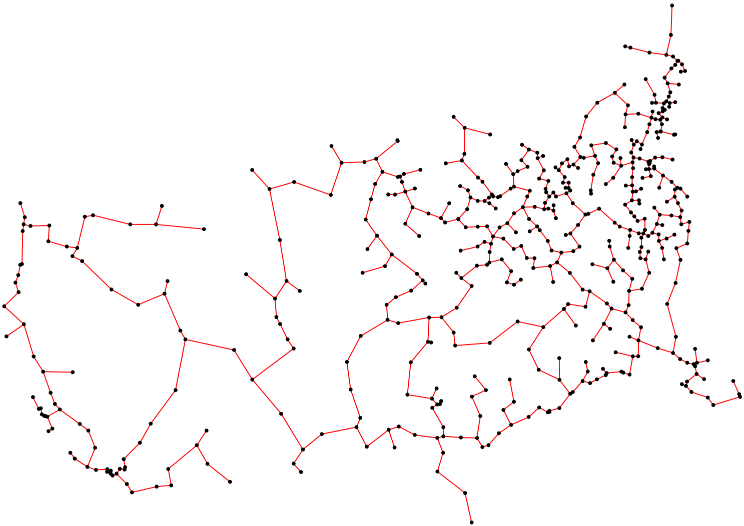
$(1+\varepsilon)$-Spanners approximate the complete graphs with error ε.

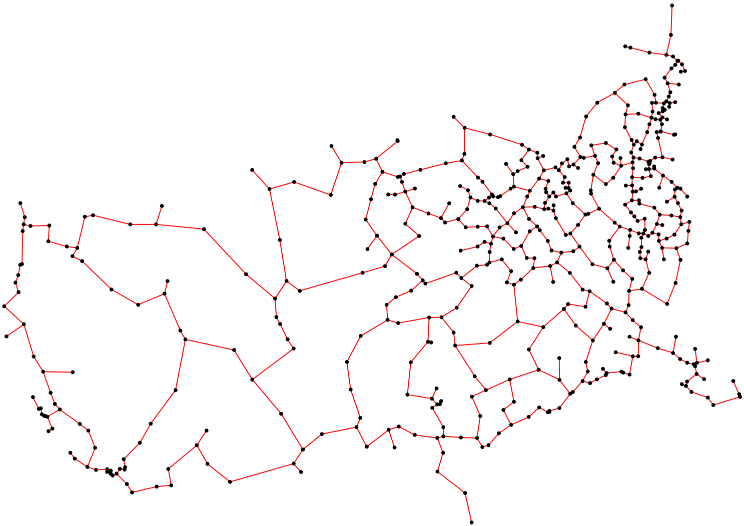


Example



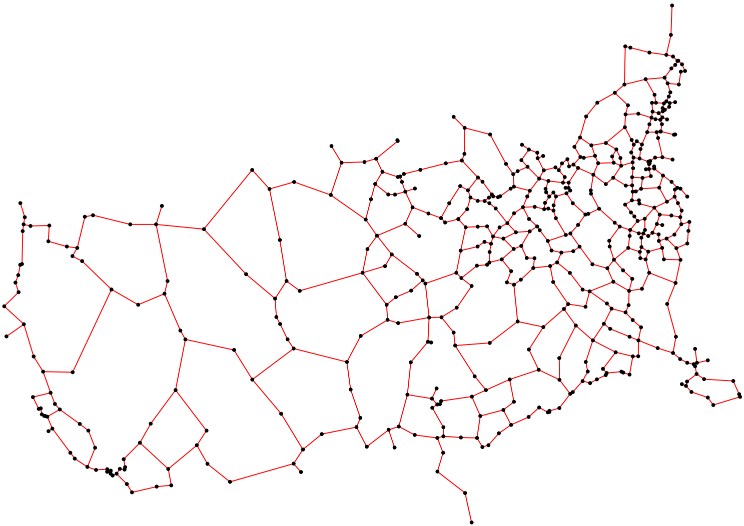
10-spanner for 532 US-cities

Example



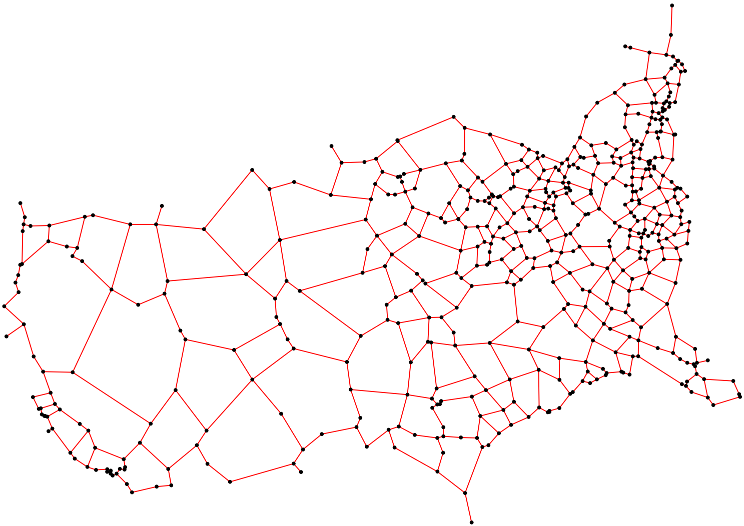
5-spanner for 532 US-cities

Example



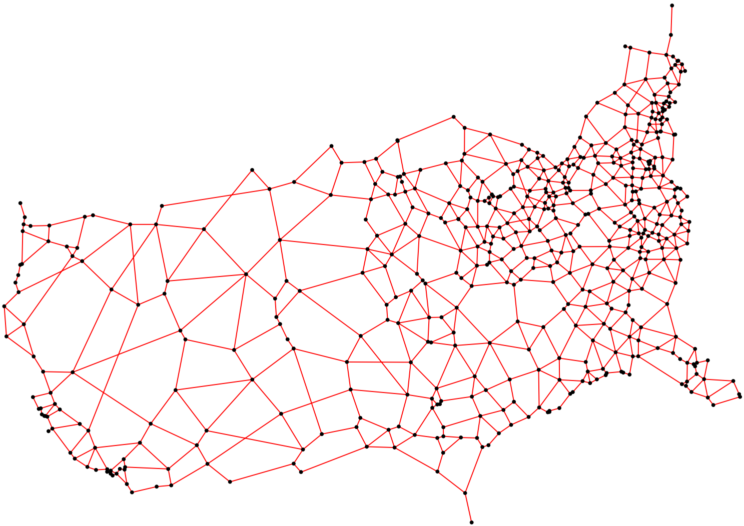
3-spanner for 532 US-cities

Example



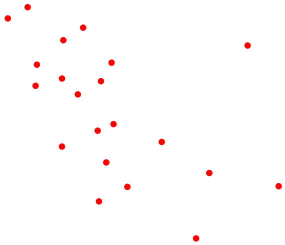
2-spanner for 532 US-cities

Example

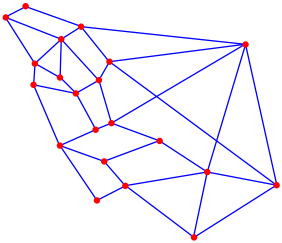


1.5-spanner for 532 US-cities

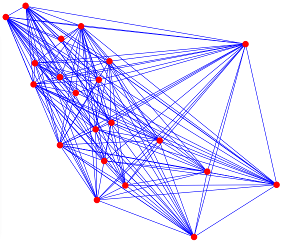
How to compute a good spanner?



Given a set V and $t > 1$



Sparse t-Spanner



Quality measurement:

- Number of edges (size)
- Weight (compared with MST)
- Maximum degree
- Diameter

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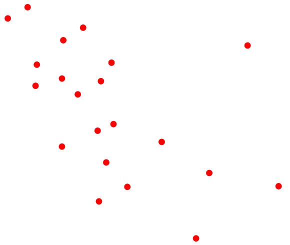

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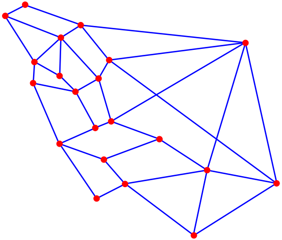

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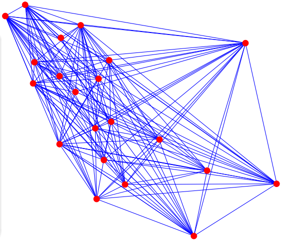

Quality measurement:

- Number of edges (size)
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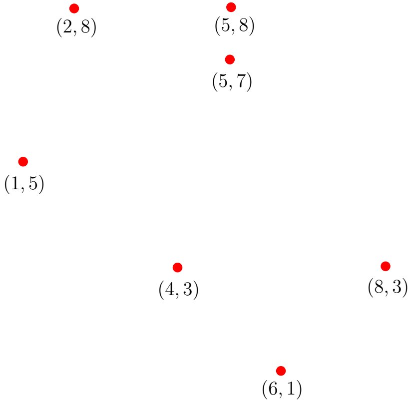
How to compute a good spanner?

Constructing sparse t-spanners:

- Greedy (Bern (1989) and Althöfer et al. (1993)).
- Θ-graph (Clarkson (1987) and Keil (1988)).
- Ordered Θ-graph (Bose et. al. (2004)).
- Well-Separated Pair Decomposition (Arya et. al. (1995)).

(Org.) Greedy Algorithm



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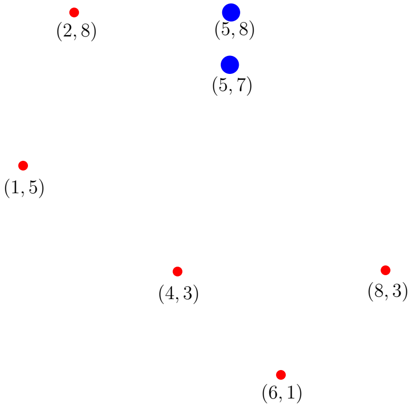

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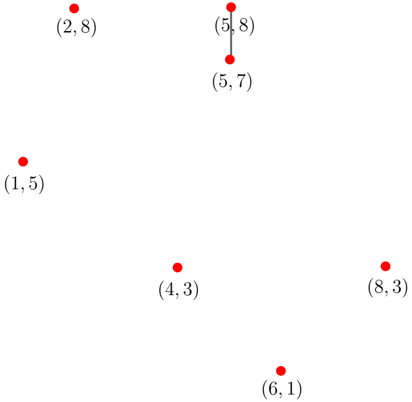

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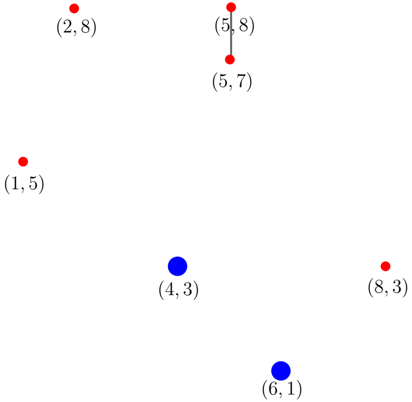

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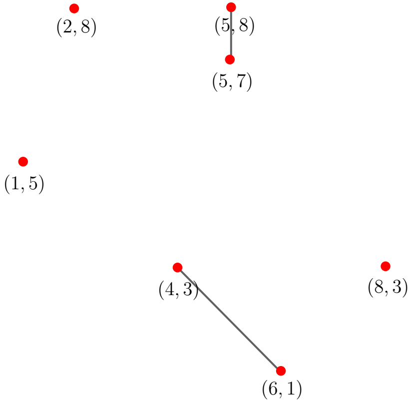

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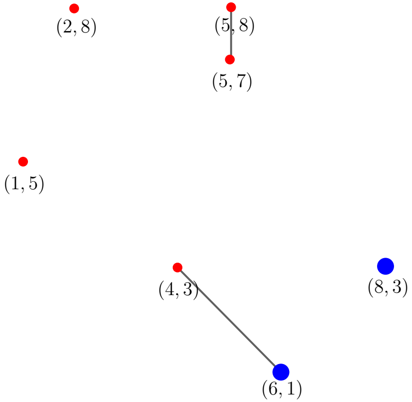

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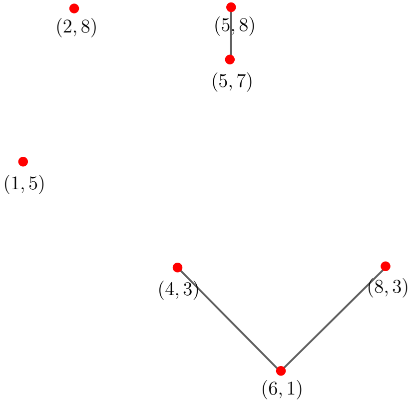

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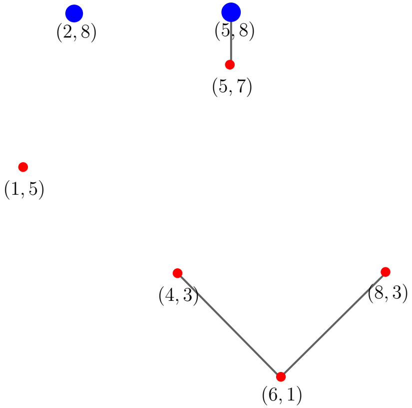

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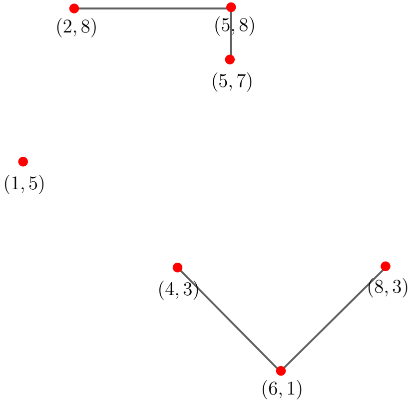

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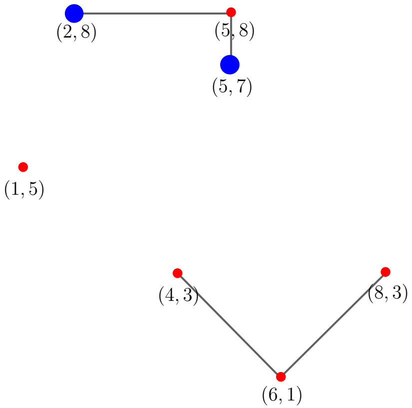

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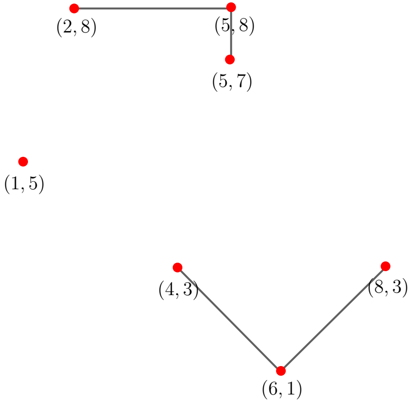

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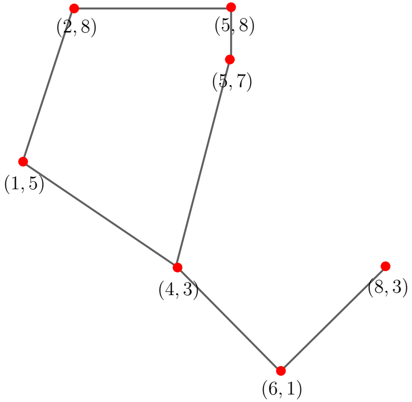

Org. Greedy

Input: V and $t > 1$
Output: t-spanner $G(V, E)$
Sort pairs of points by non-decreasing order of distance;
$E := \emptyset$;
$G := (V, E)$;
for *each pair* (u, v) *of points (in sorted order)* **do**
  if ShortestPath$(G, u, v) > t \cdot |uv|$ **then**
    Add (u, v) to E;
  end
end
return $G(V, E)$;

Time Complexity: $\mathcal{O}(n^3 \log n)$.

Storage Complexity: $\mathcal{O}(n^2)$.

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Observations:

- We only want to know if there is a t-path between u and v.
- The graph is only updated $\mathcal{O}(n)$ times.

Imp. Greedy Algorithm

Imp. Greedy

Input: V and $t > 1$
Output: t-spanner $G(V, E)$
for *each pair* $(u, v) \in V^2$ **do** Set *Weight*$(u, v) := \infty$;
Sort pairs of points by non-decreasing order of distance;
$E := \emptyset$; $G := (V, E)$;
for *each pair* (u, v) *of points (in sorted order)* **do**
    if *Weight*$(u, v) \leq t \cdot |uv|$ **then**
        Skip (u, v);
    else
        Compute single source shortest path with source u;
        for *each* w **do** update *Weight*(u, w) and *Weight*(w, u);
        if *Weight*$(u, v) \leq t \cdot |uv|$ **then** Skip (u, v);
        else Add (u, v) to E;
    end
end
return $G(V, E)$;

Imp. Greedy Algorithm

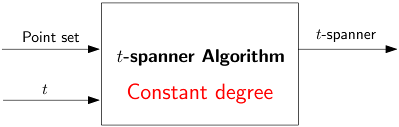
Conjecture:

The running time of Imp. Greedy is $\mathcal{O}(n^2 \log n)$.

Bose, Carmi, Farshi, Maheshvari and Smid (2008)

- The conjecture is wrong!
- They presented an algorithm which computes the greedy spanner in $\mathcal{O}(n^2 \log n)$ time (even for points from some metric spaces).

Apx. Greedy Algorithm



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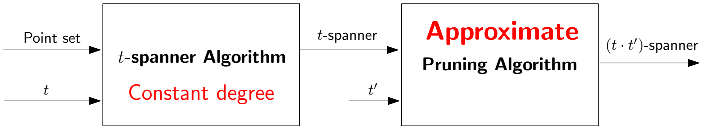

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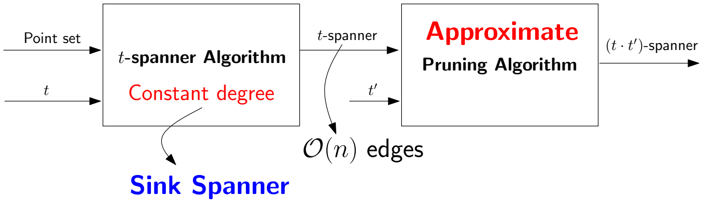

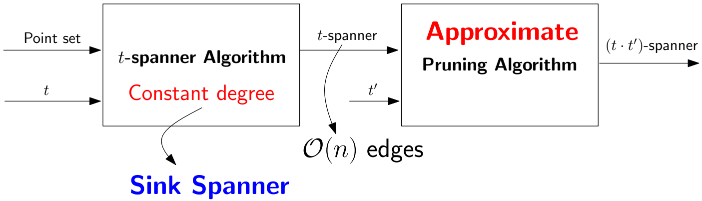

Time Complexity: $\mathcal{O}(n \log^2 n)$

Storage Complexity: $\mathcal{O}(n)$.

Θ-Graph Algorithm

$t = 3, \Theta = \pi/6$

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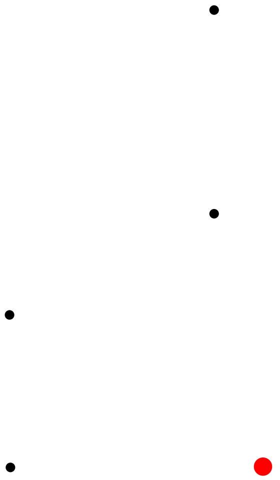

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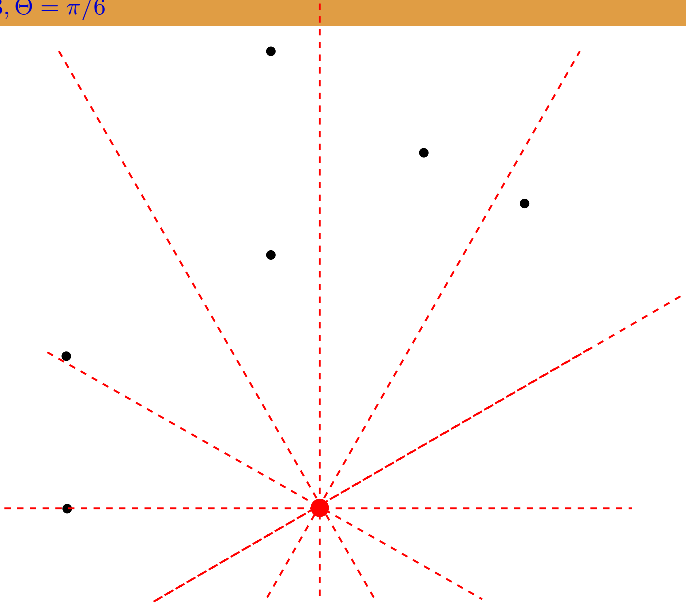

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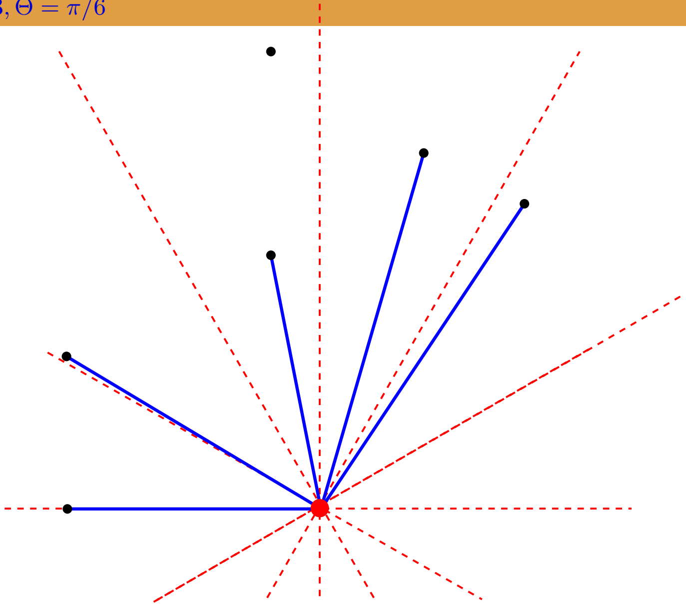

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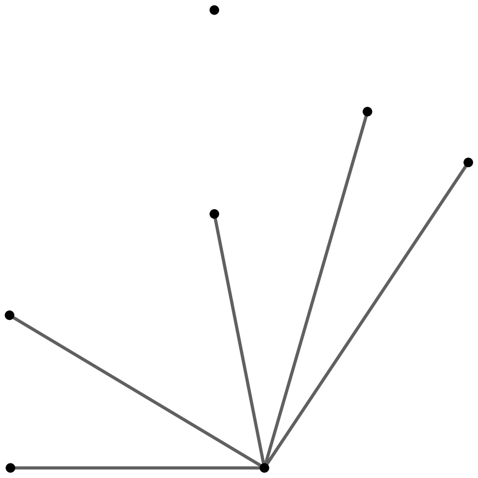

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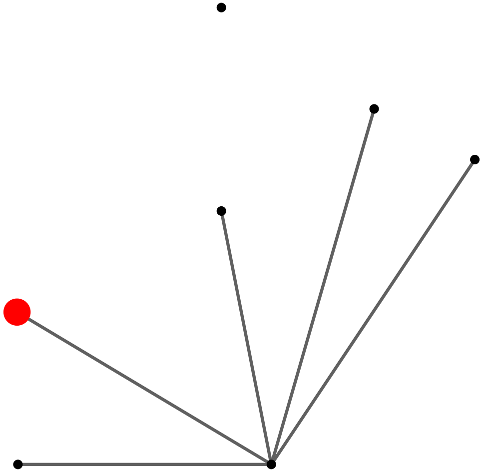

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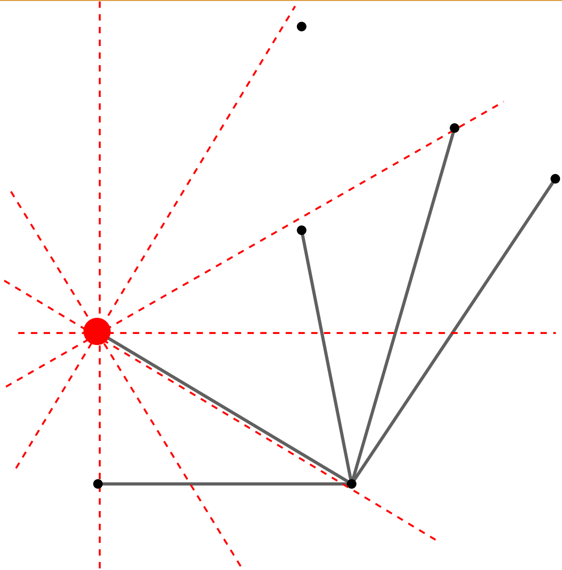

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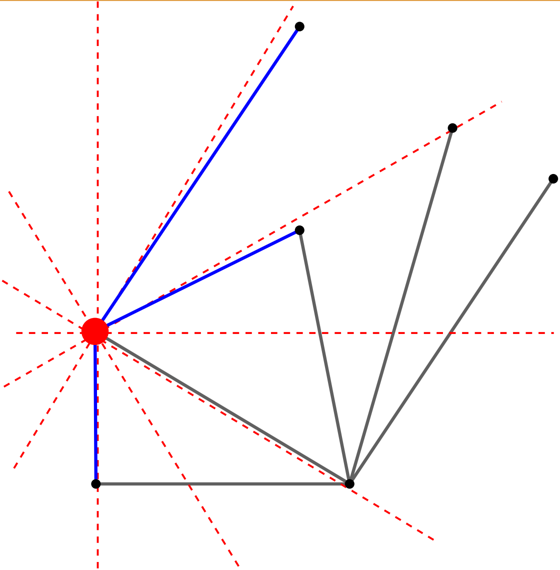

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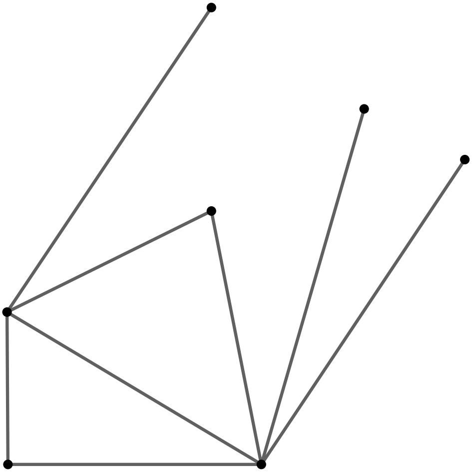

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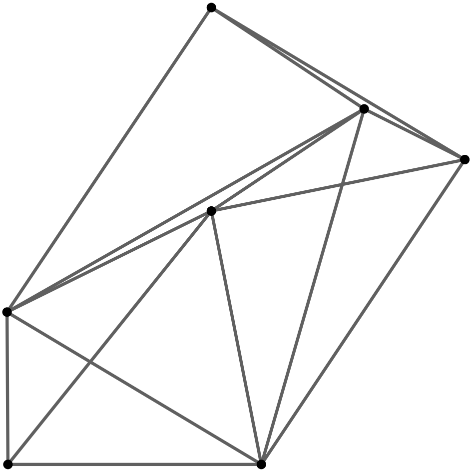

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$E := \emptyset$;

for *each point* $u \in V$ **do**

  $C_1, \ldots, C_k :=$ non-overlapping cones with angle θ and with apex at u;

  for *each cone* C_i **do**

    Connect u to the closest point in C_i;

  end

end

return $G(V, E)$;

Time Complexity: $\mathcal{O}(n \log n)$.

Storage Complexity: $\mathcal{O}(n)$.

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Time Complexity: $\mathcal{O}(n \log n)$.

Storage Complexity: $\mathcal{O}(n)$.

Variants of Θ-Graph Algorithm

Ordered Θ-Graph– $\mathcal{O}(\log n)$ maximum degree

Same as the Θ-graph algorithm, except we add points one by one in a special order.

Random Ordered Θ-Graph– $\mathcal{O}(\log n)$ spanner diameter

We add points one by one in a random order.

Sink Spanner– bounded degree

Decrease the degree of nodes by replacing some edges by paths within other nodes.

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Decrease the diameter of Θ-graph by adding some extra edges.

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Sink Spanner

A variant of Θ-graph with bounded degree

Input: V and $t > 1$
Output: t-spanner $G(V, E)$
Construct a directed $\sqrt{t}$-spanner $\overrightarrow{G}$ with bounded out-degree;
for *each point* $q \in V$ **do**
  Replace the "star" pointing to q by a $\sqrt{t}$-q-sink spanner
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A variant of Θ-graph with $\mathcal{O}(\log n)$ spanner diameter

Input: V and $t > 1$
Output: t-spanner $G(V, E)$
Set $V_0 := V$; $i := 1$;
while $V_{i-1} \neq \emptyset$ **do**
  V_i contains each points of V_{i-1} with probability $1/2$;
end
for *each* i **do**
  Construct a t-spanner $G_i(V_i, E_i)$ using the Θ-graph algorithm;
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$E = \cup_i E_i$;
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Geometric Spanner Networks

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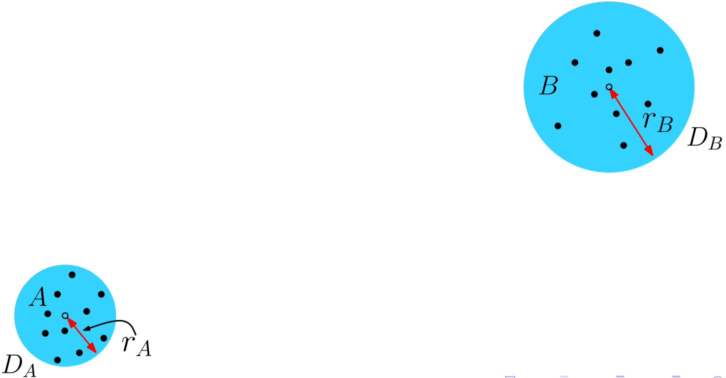

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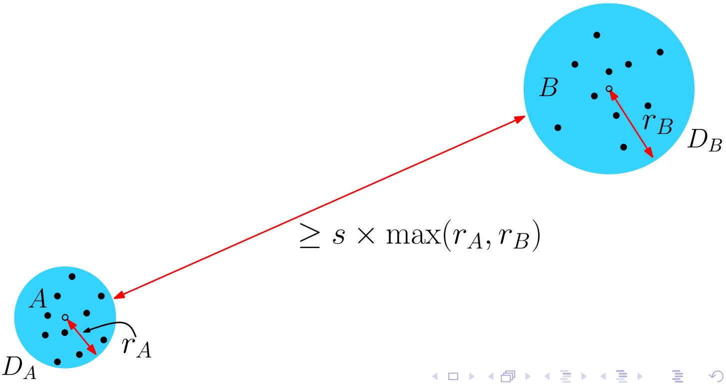

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Let $V \subset \mathbb{R}^d$ and $s > 0$. A WSPD for V with respect to s is a set $\{(A_i, B_i)\}_{i=1}^{m}$ of pairs of non-empty subsets of V such that

- $\forall i$, A_i and B_i are s-well separated,
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m : Size of WSPD.

Callahan & Kosaraju (1995)

For each set of n points, we can construct a WSPD of size $O(s^d \cdot n)$ in $O(n \log n)$ time using $O(s^d \cdot n)$ space.

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WSPD-based Algorithm

WSPD Algorithm

Input: V and $t > 1$
Output: t-spanner $G(V, E)$
Set $\mathcal{W} :=$ WSPD of V w.r.t. $s := \frac{4(t+1)}{t-1}$;
Set $E = \emptyset$;
for *each* $(A_i, B_i) \in \mathcal{W}$ **do**
    Select an arbitrary node $u \in A_i$ and an arbitrary node $v \in B_i$;
    Add edge (u, v) to E.
end
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Time Complexity: $\mathcal{O}(n \log n)$.
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-	Size	Weight	Degree	Time
Greedy spanner	$\mathcal{O}(n)$	$\mathcal{O}(wt(\mathrm{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n^2 \log n)$
Apx. greedy spanner	$\mathcal{O}(n)$	$\mathcal{O}(wt(\mathrm{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n \log n)$
Θ-graph	$\mathcal{O}(n)$	$\Theta(n \cdot wt(\mathrm{MST}))$	$\Theta(n)$	$\mathcal{O}(n \log n)$
O. Θ-graph	$\mathcal{O}(n)$	$\mathcal{O}(n \cdot wt(\mathrm{MST}))$	$\mathcal{O}(\log n)$	$\mathcal{O}(n \log n)$
WSPD spanner	$\mathcal{O}(n)$	$\mathcal{O}(\log n \cdot wt(\mathrm{MST}))$	$\Theta(n)$	$\mathcal{O}(n \log n)$
Sink-spanner	$\mathcal{O}(n)$	$\mathcal{O}(n \cdot wt(\mathrm{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n \log n)$
Skip-list spanner	$\mathcal{O}(n)^*$	$\Theta(n \cdot wt(\mathrm{MST}))^*$	$\Theta(n)$	$\mathcal{O}(n \log n)^*$

(*): Expected with high probability

Applications

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Designing approximation algorithms with spanners

Traveling Salesperson Problem (TSP)

Find the shortest tour that visits each point exactly once and return to the starting point.

Known results:

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Designing approximation algorithms with spanners

Traveling Salesperson Problem (TSP)

Find the shortest tour that visits each point exactly once and return to the starting point.

Known results:

- The problem is NP-hard even in $\mathbb{R}^d$.
- A 2-approximation algorithm for metric spaces by Rosenkrantz *et al.* (1977).
- A 1.5-approximation algorithm by Christofides *et al.* (1976).
- A PTAS ($(1+\varepsilon)$-approx. Alg.) for geometric case by Arora (1998) and Mitchell (1999).
- A PTAS for geometric case using spanners by Rao and Smith (1998).

Definition:

If G is a graph with vertex set P, then a tour of P in G is a (possibly non-simple) cycle in G that visits each point of P at least once.

Observation:

For any t-spanner G for P, there is a tour of P in G, whose weight is at most $t \cdot wt(\mathrm{TSP}(P))$.

Theorem (Rao and Smith, 1998)

Given a $(1+\varepsilon)$-spanner of a set of n points with $\mathcal{O}(n)$ size and $\mathcal{O}(wt(\mathrm{MST}))$ weight, we can compute a $(1+\varepsilon)$-approximation of $\mathrm{TSP}(P)$ in $\mathcal{O}(n \log n)$ time.

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S. B. Rao and W. D. Smith, **Approximating Geometrical Graphs via "Spanners" and "Banyans"**, STOC'98, pp. 540–550, 1998.

Applications
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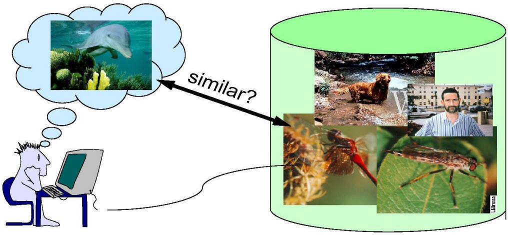

Approximate proximity searching:

- Multimedia information retrieval,
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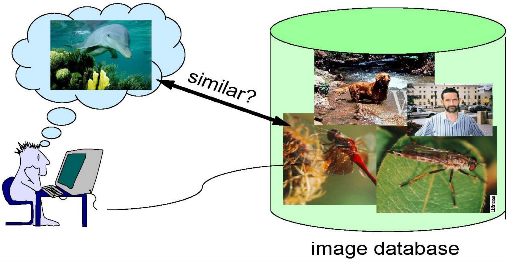

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- A meter show the similarity between any two objects.
- But evaluating the distances are expensive.
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G. Navarro, R. Paredes, and E. Chávez, **t-Spanners for metric space searching**, Data & Knowledge Engineering, pp. 820-854, 2007.

D. Russel and L. Guibas, **Exploring Protein Folding Trajectories Using Geometric Spanners**, Pacific Symposium on Biocomputing, pp. 40-51, 2005.

Current and Future Works:

- Dynamic spanners (insert and remove nodes).
- Kinetic spanners (when points move and we want to maintain an spanner all the time).
- Fault-tolerant spanners (vertex/edge fault tolerant or region fault tolerant).
- Spanners among obstacles.
- Optimization problems.
- External memory (I/O efficient) algorithms for generating spanners.
- Experimental works on spanner algorithms.

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